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Obtaining the optimal fuel conserving investment mix: a linear programming-hedonic technique approach

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OBTAINING THE OPTIMAL FUEL CONSERVING INVESTMENT MIX: A
LINEAR PROGRAMMING-HEDONIC TECHNIQUE APPROACH

Iowa State University

Ph.D. 1984

**University
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Obtaining the optimal fuel conserving investment mix:

A linear programming-hedonic technique approach

by

Terry Marie Dinan

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CHAPTER 1. INTRODUCTION

Homebuilders today face a multitude of fuel saving devices which may be incorporated into newly constructed homes. High levels of insulation, reduced air changes per hour, passive solar applications, and high energy efficiency furnaces may all be included in housing designs. Each of these investments will increase the fixed cost of building the home and decrease the fuel expenditures necessary to maintain the home at the desired temperature level. The questions addressed in this study are: What mix of these fuel saving investments is most efficient? How does the housing market evaluate energy efficiency? Does the "efficient" investment mix change as the period of time the home is to be owned is altered?

In this paper, "efficient" or "optimal" is defined as the fuel saving investment mix which maintains the desired temperature level at a minimum cost to the homeowner. Each homeowner rationally plans to minimize the cost of heating the home over the period which he owns it. Is the efficient investment mix for an individual who plans to own his home for six years (the average time that a home in Des Moines, Iowa is owned for) different than the efficient investment mix for an individual who plans to own his home for 50 years? The answer to this question, depends on how the housing market evaluates energy efficiency, i.e., the resale value of the fuel saving investments. If the market does not fully reflect future expected savings due to fuel saving investments, then the "optimal" fuel saving investment mix for an individual which

plans to own his home for a six-year period will indicate an underinvestment in conservation relative to the "optimal" investment mix for an individual who plans to own his home for a 50-year period.

The objectives of this study are to: (1) determine how energy efficiency affects the resale value of homes; (2) use this information concerning the "implicit price" of energy efficiency to estimate the resale value of fuel saving investments; and (3) incorporate these resale values into the investment decision and determine the efficient investment mix for a homeowner that plans to own a given home for three alternative time periods.

Two models are used to accomplish these objectives. A hedonic price model is used to determine the impact of energy efficiency on housing prices. The hedonic technique is used to attach implicit prices to characteristics which are not themselves bought and sold in markets, but are components of market goods. The hedonic model constructed in this study provides an estimate of the implicit price which is paid for an increase in energy efficiency in homes on the Des Moines housing market. Given this implicit price, and the efficiency of fuel saving investments (such as insulation, passive solar applications, and high energy efficiency furnaces), the resale value of fuel saving investments are estimated.

To determine how the length of time the home is to be owned affects the "optimal" investment mix, a linear programming model is used to determine the cost minimizing investment mix for a baseline house under the assumption that it will be owned for 6, 20, and 50 years. For

convenience, it is assumed that the expected life of investments in insulation, passive solar, and tight construction is 50 years. The expected life of a furnace is assumed to be 20 years. When the home is to be owned for six or 20 years, therefore, the "resale" or salvage value of each investment must be considered in the investment decision. The private homeowner who plans to own his home for a six or 20 year period will wish to weigh the incremental cost of each investment against the marginal benefits it will bring about during the time the home is owned and its resale value (Johnson, 1981):

$$C \leq \int_0^n MB(t)e^{-rt} dt + (P_2 - P_1)e^{-rn} \quad (1.1)$$

where: C = the incremental cost of the fuel saving investment,

$MB(t)$ = the marginal benefits at time t ,

r = the household's discount rate,

P_2 = market price of house with the fuel saving investment installed,

P_1 = market price of house without the fuel saving investment installed, and

n = the number of years until the owner will sell the home.

For each investment, the resale value estimated from the implicit price of efficiency obtained from the hedonic price model is used as an estimate of $(P_2 - P_1)e^{-rn}$. Information on these resale values allows an efficient investment mix to be determined when the house is to be owned for a period less than the life of the fuel saving investments.

In the 50 year linear programming model, the additional construction cost of each investment is compared to the present discounted value of the entire flow of benefits due to that investment (Isakson, 1983):

$$C \leq \int_0^T MB(t)e^{-rt} dt \quad (1.2)$$

where: T = the expected life of the fuel saving investment.

By approaching the issue of determining an efficient fuel saving investment mix in this way: (1) the impact of energy efficiency on housing prices is examined; (2) a method of attaching resale values to fuel saving investments is determined; (3) using the resale values obtained, an optimal investment mix may be determined under the assumption that the house is to be resold after a period of years less than the life of the fuel saving investments; and (4) the "efficient" investment mixes obtained under the assumption that the home is to be owned for 6, 20, and 50 years, alternatively, may be compared.

CHAPTER 2. THEORETICAL MODEL

The hedonic technique is used to attach resale values to fuel saving investments, and a linear programming model is used to determine the "optimal" fuel saving investment mix. By choosing to use a cost minimizing linear programming model to determine the "optimal" fuel saving investment mix, cost minimization was defined as the relevant criteria for selecting among alternative investments. These criteria were chosen over a utility maximization approach. In this chapter, the theoretical rationale for choosing a cost minimization criteria is explored, and the theoretical development and foundation of the hedonic model are discussed. A "new approach" to consumer theory offered by Lancaster in 1966 provides a theoretical basis for both the use of the cost minimizing linear programming model and the hedonic pricing model.

First, Lancaster's model and its extension, the household production function model, will be developed. In the second section, the theoretical justification they provide for the cost minimizing linear programming model will be examined. The basis which Lancaster's approach to consumer theory provides for the hedonic price model is discussed in the third section. Finally, the necessary assumptions and the theoretical underpinnings of the hedonic model will be examined. This discussion will focus on the work of Tiebout, Rosen, and Freeman.

Discussion of Lancaster's Approach to Consumer
Theory, the Household Production Model, and
Their Relevance to This Study

Lancaster's "new approach" to consumer theory involved redefining the utility function. Traditional consumer theory defined utility as a function of commodities, i.e., $U = U(q_1, q_2, \dots, q_n)$ where q_1, \dots, q_n is a vector of market goods. Lancaster's new approach defined utility as a function of good characteristics, rather than of the market goods themselves. Utility may, therefore, be defined as:

$$U = U(Z_1, Z_2, \dots, Z_n) \quad (2.1)$$

where: Z_j = the total amount of characteristic j obtained by the consumer.

Since utility is a function of characteristics, market goods are demanded only because of the characteristics they possess. Lancaster assumed that the vector of characteristics is related to the quantities of market goods consumed by a linear consumption technology:

$$Z_j = \sum_{i=1}^n B_{ij} q_i \quad (2.2)$$

where: q_i = the quantity of good i consumed and

B_{ij} = the amount of characteristic j found in good i .

Consumers choose quantities of market goods to maximize utility subject to a budget constraint and the consumption technology available:

$$\begin{aligned}
 &\text{Max } U(Z), & (2.3) \\
 &\text{S.T. } Z = Bq, \\
 &\quad Y \geq Pq, \\
 &\quad Z, q \geq 0,
 \end{aligned}$$

where: Z = the vector $[Z_j]$,
 B = the matrix $[B_{ij}]$,
 Y = the consumer's income,
 q = the vector $[q_i]$, and
 P = the vector of prices, $[P_i]$.

This utility maximization procedure yields an optimal bundle of characteristics, Z^* . Lancaster described three cases which may occur in this utility maximizing process. In the first case, the number of characteristics which provide utility outnumbers the number of market goods available. In the second case, the number of characteristics equals the number of market goods available. In the third case, the number of market goods available outnumbers the number of characteristics which enter the individual's utility function. Lancaster asserted that case three is the situation which most likely typifies the U.S. In this case, in which the number of market goods outnumbers the number of characteristics desired, the consumer will wish to purchase the mix of market goods which provides the optimal bundle of characteristics at a minimum cost. The most efficient method of obtaining this optimal characteristics bundle is determined by the following model:

$$\begin{aligned}
 &\text{Min } Pq, & (2.4) \\
 &\text{S.T. } Bq \geq Z^*, \\
 &\quad q \geq 0,
 \end{aligned}$$

Lancaster's theory was expanded upon by proponents of the household production function model. The household production function model incorporates the role of the household's time in obtaining the arguments of the utility function. The household production function approach views the direct arguments of the consumer's utility function (the Z_j 's in equation 2.1) as commodities which are produced by the household itself. These "commodities" are needs of the household such as warmth, entertainment, and well-groomed hair. The household's commodity production process is an activity of combining purchased market goods and services with some of the household's own time. Viewed in this framework market goods do not yield utility directly, but are inputs used in the household's production process (Michael, 1972). The consumer's demand for market goods is a derived demand, analogous to the derived demand for a factor of production by a firm. For example, the consumer uses market goods (such as furnaces, natural gas, and solar applications) and time, to produce the basic need of warmth. The commodity warmth is the direct argument in the household production function, rather than the market goods which may be used to provide warmth.

The household production function approach to consumer theory is similar to Lancaster's approach. In both approaches, market goods are

not the direct arguments in the consumer's utility function; rather, the demand for market goods is a derived demand. The household production model differs from Lancaster's approach in that the direct arguments of the consumer's utility function are assumed to be "commodities" produced by the household, rather than good characteristics. The household uses its own time and market inputs to produce the "commodities." For example, the household uses its time and a washing machine to produce the commodity clean clothes.

The household production model terminology will be used in the remainder of this analysis. The term commodities will be used in a basic sense to refer to the direct arguments in the household's utility function, e.g., clean clothes, well groomed hair, and warmth. Market goods will be used to refer to the goods which can be used to produce the desired commodities. Washing machines, haircuts, and furnaces are examples of market goods which can be used to produce clean clothes, well groomed hair and warmth. The demand for these market goods is a derived demand. Commodities are produced by the household according to the production function:

$$Z_j = f_j(x_i, t_j; H) \quad (2.5)$$

where: x_i = a vector of market goods,

t_j = household's time input in the production of Z_j , and

H = household's available quantity of some environmental variable.

The household seeks to maximize its utility function subject to its money income constraint:

$$Y_m = wt_m + V = \sum_{i=1}^n x_i P_i \quad (2.6)$$

where: w = wage rate in the labor market,

t_m = time spent in labor market, and

V = nonwage income for the period,

and a time constraint:

$$t = \sum_{j=1}^n t_j + t_m. \quad (2.7)$$

"In this framework, the household is viewed as a small firm producing many products, from which it derives utility" (Michael, 1972). The household has a demand function for each commodity:

$$Z_j^d = d_j\left(\frac{Y}{\pi}, \frac{\pi_j}{\pi}\right) \quad (2.8)$$

where: π_j = average price of Z_j .

π = price level = $\pi_1 s_1, \pi_2 s_2, \dots, \pi_n s_n$,

where s_j = expenditure share on commodity j .

As previously stated, one commodity, or characteristic, which enters the household's utility function is the warmth that is experienced in the home during the heating season. The warmth that the household experiences is a choice variable. As shown in equation (2.8), the level

of warmth that the household chooses is a function of its real income and the relative price of warmth. When choosing the level of warmth, the household considers the tradeoff between warmth and the other arguments of its utility function. To the extent that increased warmth involves higher expenditures (e.g., for fuel, insulation, additional south glass), an increase in the level of warmth which is chosen means that the household has less funds to devote to the production of the other commodities which enter its utility function (e.g., entertainment, attractiveness of the home). To the extent that an increase in warmth involves additional time spent by the household (e.g., to install insulation, caulk windows), a higher level of warmth means that the household has less time available for the production of other commodities which enter its utility function.

The warmth in the home is a function primarily of the internal temperature of the home. Additional clothes, blankets, and space heaters may be used, to a limited degree, as a substitute for a higher internal temperature. As the price of fuels, insulation, glass and electricity increase, the relative price of a high internal temperature increases. The household may choose to maintain the same temperature and devote an increased share of its income to the production of heat, or it may choose to maintain a lower temperature level and substitute increased clothing and blankets for the heat forgone.

Once the utility maximizing level of warmth is chosen, and the internal temperature level necessary to attain the desired warmth is determined, the method of obtaining this temperature level must be

established. In this study, cost minimization is selected as the criterion for choosing the method of maintaining the desired temperature level. Purchasing natural gas, buying high efficiency furnaces, adding insulation, reducing air changes per hour in the home, and using solar applications are all possible energy providing investments. A linear programming model is used to select the mix of these investments which will maintain the desired temperature level at a minimum cost. The following section will discuss the use of a cost minimizing linear programming model to select the investment mix.

Justification of the Use of a Cost Minimizing Linear Programming Model to Choose Among Alternative Fuel Saving Investments

The household chooses the level of warmth in the home, Z_j^* , which maximizes its utility function (equation 2.1). There are numerous methods by which the household may produce this level of warmth. Purchasing natural gas, electricity, insulation, south glass, high efficiency furnaces, and decreasing the air changes per hour in the home are all methods of producing the utility maximizing level of warmth in the home. Since there are many market goods which may be purchased to produce warmth, the situation is analogous to Lancaster's case three, where there are numerous goods which may be used to obtain one characteristic. In the numerous goods case, the household seeks to obtain each characteristic in the most efficient (minimum cost) way, as described by equation (2.4). Incorporating the household's time

constraint as well as its budget constraint into the analysis, it chooses the mix of inputs which:

$$\begin{aligned}
 &\text{Min } \sum_{i=1}^n p_i x_i, & (2.9) \\
 &\text{S.T. } (1) \ x_i B_{ij} \geq Z_j^*, \\
 & & (2) \ t_j \leq t_j^*, \\
 & & (3) \text{ design preferences}
 \end{aligned}$$

where: x_i = the market inputs used to produce heat,

B_{ij} = the quantity of heat obtained from input x_i , and

t_j^* = the maximum time which is to be allocated to the production of heat.

Market inputs which are used to produce warmth may be chosen according to an efficiency criterion because the inputs which produce warmth (e.g., furnaces, insulation, solar applications) are not direct arguments of the household utility function and, therefore, are not chosen according to a utility maximizing criteria. The argument of the utility function is warmth, producing a derived demand for the inputs which may produce this utility maximizing level of warmth in the most efficient way.

Due to the fact that a cost minimizing (rather than a utility maximizing) criterion is used to obtain the optimal combination of warmth producing inputs, a linear programming model may be designed with equation (2.9) as its objective function and constraints 1-3 incorporated into the model design.

A limitation of the linear programming model is that it does not pick the utility maximizing temperature level, which is derived by the utility maximizing process described above. If two households desire different internal temperature levels, then the cost minimizing investment mix may be different for the two homes. Sensitivity analysis is performed to determine how the optimal fuel saving investment mix is altered when the home is maintained at alternative temperature levels.

A strength of the linear programming model is that it is able to choose the most efficient means of producing the desired temperature level subject to constraints on the other arguments of the household's utility function. For example, an argument of the utility function may be the amount of daylight in the home. This may be accounted for in the linear programming model by placing a constraint on the amount of window space included in the home.

A key question in constructing the linear programming model is: What is the appropriate time horizon? To minimize long run total cost, the present discounted value of the marginal benefits would be compared to the marginal cost of each investment (equation 1.1). In this analysis the life of the insulation, tight construction, and passive solar applications are assumed to be 50 years, indicating that a 50 year planning horizon should be used in the linear programming model. In reality, however, homeowners may only expect to live in the house for n years, where n is less than 50. It is rational for these homeowners to make their investment decision based on the cost of each investment, the fuel savings that they will obtain during the n years they live in

the house, and the "return" (or terminal value) on each investment that they will receive when the house is resold (equation 1.2).

The "return" that a homeowner receives on a fuel conserving investment is the amount which this fuel saving feature increases the resale price of the home. If the resale value can be estimated for each energy efficiency increasing investment, then this knowledge can be incorporated into the linear programming model. Knowledge of the resale values of fuel saving investments will allow a cost minimizing investment mix to be obtained for a homeowner who plans to own his new home for less than 50 years. In order to calculate the resale value of fuel saving investments, the implicit price of the energy efficiency level of a home must be estimated, i.e., how does an increase in energy efficiency affect the resale value of the house? A hedonic model is used to estimate this implicit price. The following section discusses the assumptions and theoretical foundation of the hedonic model.

The Theoretical Background and Assumptions of the Hedonic Model

Freeman (1979b) described the hedonic technique as "a method for estimating the implicit price of the characteristics which differentiate closely related products in a product class" (p. 78). In this study, the hedonic technique is applied to the housing market. Houses are differentiated by their size, number of rooms, location, quality of construction, energy efficiency, and numerous other structural and neighborhood characteristics. The hedonic technique is used to determine

the effect that each of these characteristics has on the selling price of the house. A basic assumption of the hedonic model is that each house may be described by a vector of characteristics:

$$H = (h_1, h_2, \dots, h_n) \quad (2.10)$$

where: (h_1, h_2, \dots, h_n) is a vector of characteristics of the house. The price of each house may then be written as a function of the price determining characteristics:

$$P(H) = P(h_1, h_2, \dots, h_n) \quad (2.11)$$

where: $P(H)$ = the selling price of the house.

By examining a large number of houses having various combinations of these characteristics, it is possible to obtain the form of the functional relationship between the price of the house and the vector of price determining characteristics. If the function relating the price of the house to its characteristics can be identified, then the implicit price associated with any given characteristic can be determined by differentiating the function with respect to that characteristic, holding all other factors constant, i.e.:

$$\frac{\partial P(H)}{\partial h_i} = P_{hi} \quad (2.12)$$

P_{hi} is the implicit price associated with characteristic h_i . It represents the increase in $P(H)$ that an individual must pay to obtain one more unit of housing characteristic h_i . For example, if h_i was the number of bedrooms in the house, P_{hi} would represent the additional price a consumer must pay for a house having three rather than two bedrooms.

Lancaster's approach to consumer theory provides a basis for the hedonic model. As was previously discussed, Lancaster defined utility as a function of characteristics, rather than market goods (see equation 2.1). Goods, therefore, are desired for the characteristics they possess. It follows logically that goods may be described by a vector of characteristics, as they are in a hedonic price model (equation 2.10). The concept of goods possessing implicit or shadow prices may be found in Lancaster's work. Recall that when there are more market goods available than characteristics desired (Lancaster's case three), the consumer chooses the most efficient combination of market goods to obtain his optimal characteristic bundle (equation 2.4). The dual of this cost minimization process is:

$$\text{Max } \rho Z^*, \quad (2.13)$$

$$\text{S.T. } \rho B \leq P,$$

where: ρ are the shadow prices of the characteristics.

Note that when the constraint to this maximization process holds, $P = \rho B$. Therefore, the price of the good may be written as a function of its characteristics and the implicit price of a characteristic, ρ_i , may be found by taking the partial derivative of the price with respect to the characteristic, B_i . This is the process used in a hedonic price model.

As demonstrated, Lancaster's work provides a foundation for the hedonic technique. Tiebout, Rosen, and Freeman have further examined the necessary assumptions and theoretical underpinnings of the hedonic price model. Tiebout developed a theory of local expenditures. The basic concept of the local expenditure model and several of the underlying assumptions are similar to those of the hedonic model. The fundamental concept of the Tiebout model is that consumers express their demand for locally provided public goods by their choice of the community in which they live. "The consumer-voter may be viewed as picking the community which best satisfies his preference pattern for public goods" (Tiebout, 1956, p. 418). Similarly, in a hedonic model it is assumed that an individual chooses a home which best represents his preference pattern for structural and locational characteristics.

Several of the assumptions made in the Tiebout local government model are applicable to the hedonic price model. The Tiebout model assumes that consumer-voters have full knowledge of differences among revenue and expenditure patterns and react to these differences. Similarly, in the hedonic model of the housing market one must assume that individuals have full knowledge about the differences in

characteristics among homes, and that the price they offer for a home is a function of these characteristics. If a buyer lacks knowledge of a specific housing characteristic, then the price that he is willing to offer for the house will not reflect his demand for that characteristic. For example, if an individual was unaware of the energy efficiency of a house, then energy efficiency may not be considered a relevant variable in determining his offer price for the house. Since the hedonic price model is based on the assumption that the selling price of a house reflects the supply and demand of the individual characteristics of the home, the assumption of full knowledge of those characteristics is crucial.

Tiebout also assumed that there are a large number of communities in which consumer-voters may choose to live. Similarly, in the hedonic price model it must be assumed that the consumer is able to choose from a large number of homes having differing characteristics. If this assumption did not hold then buyers would not be able to find a home which fit their preference pattern. In this case, it could not be assumed that the price they offer for the home reflects their desire for all of the housing characteristics. For example, assume that a buyer wishes to purchase a home having a swimming pool but does not wish to pay a premium for energy efficiency. Also, assume that there is only one home available having a swimming pool, and that this house is energy efficient. The buyer must purchase this home in order to obtain the swimming pool, however, the house is actually more energy efficient than he desires. The price he offers for the house, therefore, reflects his

desire for the the swimming pool but not his demand for energy efficiency. To assume that the selling price of each home reflects the buyer's demand for each of the individual housing characteristics, it must be assumed that the number of homes on the market is large enough so that each buyer may find a home having the bundle of characteristics which he desires. According to Rosen (1974), it must be assumed that a sufficiently large number of houses are available so that the choice between various houses (combinations of characteristics) is continuous for all practical purposes.

In addition to having a large number of houses available, Freeman (1979b) added that "It must be assumed that the housing market is in an equilibrium, that is, that all households have made their utility-maximizing residential choice given the prices of alternative housing locations, and that these prices just clear the market given the existing stock of housing and its characteristics" (p. 122).

Rosen (1974) focused on the assumption of market equilibrium in using the hedonic technique. He defined a hedonic model as "a description of a competitive equilibrium in a plane of several dimensions on which both buyers and sellers locate" (p. 35). For a good described by n characteristics, any location on the plane is represented by a vector of coordinates $z = (z_1, z_2, \dots, z_n)$ where z_j measures the amount of the j^{th} characteristic contained in each good, and the vector (z_1, z_2, \dots, z_n) completely describes the good. In equilibrium, the amount of commodities offered by sellers at every point on the plane must equal the amount demanded by consumers choosing to locate there.

The composite goods being modeled in this study are homes on the housing market. As discussed previously, each house is described by a vector of structural and locational characteristics (equation 2.10) and the price of the house is written as a function of those characteristics (equation 2.11). Home purchasers determine their purchasing decision, and suppliers determine their supply decision according to the implicit price function, $P(H) = P(h_1, h_2, \dots, h_n)$. When consumers shop for a home, they look for the lowest priced home having all the characteristics they desire. The price function, $P(H)$, therefore, represents the minimum price of a house, given its unique set of characteristics. Since $P(H)$ is exogenous to both buyers and suppliers of homes, competition prevails. An equilibrium exists when the number of houses supplied having a given set of characteristics equals the number of houses demanded with that unique set of characteristics. Rosen examined the underlying consumption and supply decisions which lead to this market equilibrium. The next portion of this paper will discuss Rosen's description of a market equilibrium, using the example of a house as a composite good. The actual description of the competitive equilibrium is based on Rosen's article (1974), yet the notation and clarifying examples are specific to the housing market.

Consumers maximize utility and producers maximize profits subject to the exogenous implicit price function, $P(H)$. First, examine the consumption decision. Assume each consumer plans to maximize his utility function by his choice of a house. His utility function may be written as:

$$U = u(x, h_1, h_2, \dots, h_n) \quad (2.14)$$

where: x = quantity of all other goods consumed, for convenience x may be thought of as money,

$$P_x = 1,$$

and, U is assumed to be strictly concave. Income can be written in terms of x as:

$$y = x + P(H) \quad (2.15)$$

where: y = total income,

$$H = (h_1, h_2, \dots, h_n), \text{ and}$$

$P(H)$ = housing (hedonic) price function.

To maximize utility requires choosing x and (h_1, h_2, \dots, h_n) such that the budget constraint and first order conditions are satisfied:

$$L = u(x, h_1, h_2, \dots, h_n) + \lambda(y - x - P(H)), \quad (2.16)$$

$$\frac{\partial L}{\partial h_1} = u_{h_1} - \lambda \frac{\partial P}{\partial h_1} = 0,$$

$$\frac{\partial L}{\partial h_2} = u_{h_2} - \lambda \frac{\partial P}{\partial h_2} = 0,$$

⋮

$$\frac{\partial L}{\partial h_n} = u_{h_n} - \lambda \frac{\partial P}{\partial h_n} = 0,$$

$$\frac{\partial L}{\partial x} = u_x - \lambda = 0, \text{ and}$$

$$\frac{\partial L}{\partial \lambda} = y - x - P(H) = 0.$$

Simultaneously solving these equations results in choosing x and (h_1, h_2, \dots, h_n) such that:

$$\frac{U_{hi}}{U_x} = \frac{\partial P}{\partial h_i} = P_{hi}, \quad i=1,2,\dots,n. \quad (2.17)$$

The consumer maximizes utility by choosing a house which has the combination of characteristics such that equation (2.17) is satisfied. The second order conditions are fulfilled if the usual assumptions regarding U hold and if $P(H)$ is not sufficiently concave.

Rosen demonstrated the spatial context of the consumer decision by defining a bid (or value) function: $\theta(h_1, h_2, \dots, h_n; u, y)$. The bid function defines a family of indifference surfaces relating the characteristic h_i to money. It defines the maximum price a consumer is willing to spend for alternative values of (h_1, h_2, \dots, h_n) given his income and utility level, i.e., given that:

$$U(y - \theta, h_1, h_2, \dots, h_n) = u. \quad (2.18)$$

By differentiating equation (2.18) with respect to each housing characteristic we obtain:

$$U_{hi} - \theta_{hi} U_x = 0 \quad (2.19)$$

or,
$$\theta_{hi} = \frac{U_{hi}}{U_x}.$$

Assuming that each household characteristic is a "good" implies that each household characteristic has a positive marginal valuation, i.e., $U_{hi} > 0$. Likewise, U_x is greater than zero, therefore, $\theta_{hi} > 0$. θ_{hi} is the implicit marginal valuation which a consumer places on a housing characteristic, h_i , at a given utility index and income. The result $\theta_{hi} > 0$ indicates that a consumer is willing to bid more for a higher level of a housing characteristic. For example, he will bid more for a house which has four bedrooms than for one which has three bedrooms. The value function $\theta(h_1, h_2, \dots, h_n; u, y)$, therefore, is increasing in each characteristic h_i .

Differentiating equation (2.18) twice with respect to h_i , we obtain:

$$\theta_{hihi} = (U_x^2 U_{hihi} - 2U_x U_{hi} + U_{hi}^2 U_{xx}) / U_x^3 < 0. \quad (2.20)$$

Equation (2.20) reveals that the value function is changing at a decreasing rate. Together equations (2.19) and (2.20) reveal that the value function $\theta(h_1, h_2, \dots, h_n; u, y)$ is increasing in each housing characteristic, h_i , at a decreasing rate. Drawing this relationship for an individual characteristic, h_1 , in $\theta - h_1$ space, we obtain:

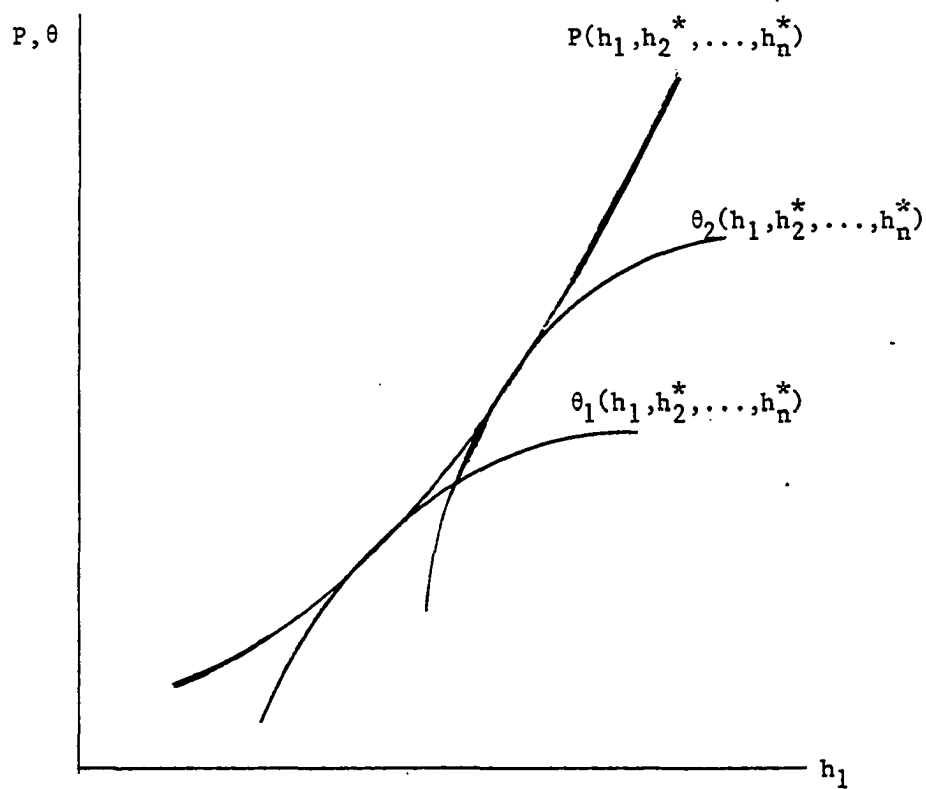


Figure 2.1. Utility maximizing consumption decision for two buyers

Recall that $P(H)$ is the minimum price the consumer must pay for each level of characteristic h_i . Since increasing the level of h_i requires additional resources, suppliers will only be willing to supply a higher level of h_i at a higher price. $P(H)$, therefore, is upward sloping, as shown in Figure 2.1. Utility is maximized when the amount the consumer is willing to pay for each housing characteristic equals the minimum amount which he must pay for it (i.e., $\theta(H; u, y) = P(H)$), and when the consumer's marginal rate of substitution between h_i and money is equal to the marginal rate of transformation between them (i.e., $\theta_{hi} = P_{hi}$). This

occurs when the consumer's value function is tangent to the price function, $P(H)$, as shown in Figure 2.1.

Figure 2.1 represents the choice of an optimal level of housing characteristic h_1 , given utility and income and given that all other housing characteristics are at their optimal levels. Let h_1 be energy efficiency. The consumer would choose the amount of energy efficiency in the home he purchases such that the marginal amount he is willing to pay for increased efficiency is equal to the minimum price that the marginal unit may be obtained for on the housing market. Two different buyers are depicted in Figure 2.1. Buyer 2 places a higher value on energy efficiency than buyer 1, therefore, he purchases a home having a higher efficiency level.

Figure 2.1 depicts an equilibrium choice of energy efficiency for two different buyers on the housing market. Each unique house (i.e., combination of characteristics) represents a point on a plane and is represented by a vector of coordinates $H = (h_1, h_2, \dots, h_n)$. Equilibrium for the market as a whole exists when demand equals supply at each point on the plane. The process by which the buyer arrives at a consumption decision has been discussed. The next key issue is the method by which the production decision is made.

Let $M(H)$ equal the number of units having specification $H = (h_1, h_2, \dots, h_n)$ produced by a firm. The total costs of the firm are given by $C(M, H; B)$, where B is a shift parameter reflecting differences in technology or factor prices among firms. Assume that C is concave with $C(0) = 0$. The marginal cost of producing more units of specification H

and the marginal cost of increasing each component in the units design, h_1, h_2, \dots, h_n , are both positive and increasing, i.e., C_m and $C_{hi} > 0$. Each firm chooses M and h_1, h_2, \dots, h_n to maximize profit, where:

$$\pi = M(P(H)) - C(M, h_1, h_2, \dots, h_n). \quad (2.21)$$

The profit maximizing model design is obtained by differentiating profit with respect to each characteristic:

$$P_{hi}(H) = \frac{C_{hi}}{M}, \quad i=1, \dots, n. \quad (2.22)$$

Differentiating the profit function with respect to M , the number of units produced having design H , yields:

$$P(H) = C_M. \quad (2.23)$$

At the optimal design, the marginal revenue from each additional characteristic equals the marginal cost of including it in the model design. The quantity of units having design H are produced up to the point where $P(H)$ equals the marginal cost of producing a unit of design H .

Rosen defined an offer function $\phi(h_1, h_2, \dots, h_n; \pi, B)$. This offer function indicates the prices a firm is willing to accept on units of various designs, assuming a constant profit level and that quantities produced of each model are optimally chosen. Now, the profit function

can be written as:

$$P = M\phi - C(M, h_1, h_2, \dots, h_n). \quad (2.24)$$

ϕ equals the offer price the seller is willing to accept for design H:
H:

$$\phi = C_m(M, h_1, h_2, \dots, h_n). \quad (2.25)$$

Since $P(H)$ is the maximum price obtainable for a model of design H, ϕ_{hi} equals the marginal reservation supply price of each attribute h_i at a constant profit:

$$\phi_{hi} = \frac{C_{hi}}{m} > 0. \quad (2.26)$$

Profit maximization and the optimum design occurs when $P(H^*) = \phi(h_1^*, h_2^*, \dots, h_n^*; \pi^*, B)$, and $P_{hi}(H^*) = \phi_{hi}(h_1^*, h_2^*, \dots, h_n^*; \pi^*, B)$. Figure 2.2 shows the offer curves of two firms for characteristic h_1 , given B, a constant profit level, and given that all other characteristics, (h_2, \dots, h_n) , are at their optimum values. As shown in Figure 2.2, firm 2, with offer curve ϕ_2 , has a comparative advantage in the production of attribute h_1 . This comparative advantage is due to a different value of B than firm 1. At the profit maximizing design, each producer's offer curve is tangent to the price function, $P(H)$.

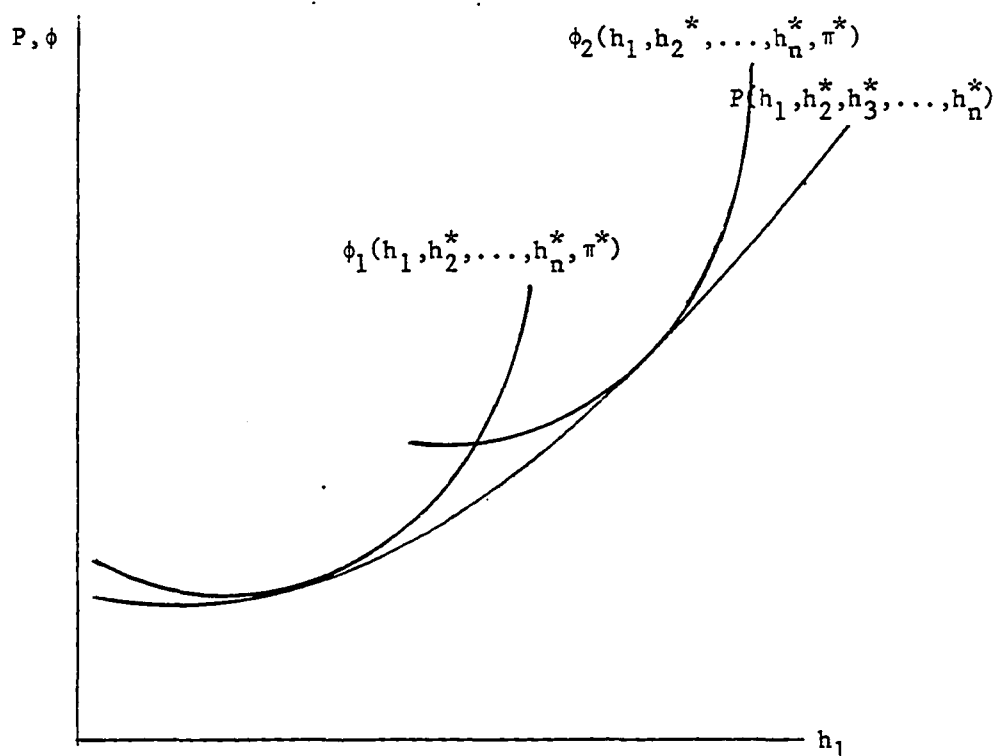


Figure 2.2. Profit maximizing production decisions for two suppliers

Joining the production and demand decisions, it can be seen that market equilibrium occurs when the offer curves are tangent to the value functions at every point (unique unit design) on the plane. This equilibrium is shown in Figure 2.3. The supplier offer functions and consumer bid (value) functions share the common gradient of the market clearing implicit price function $P(H)$. Therefore, $P(H)$ represents a joint envelope of a family of consumer value functions and a family of supplier offer functions (Rosen, 1974, p. 44).

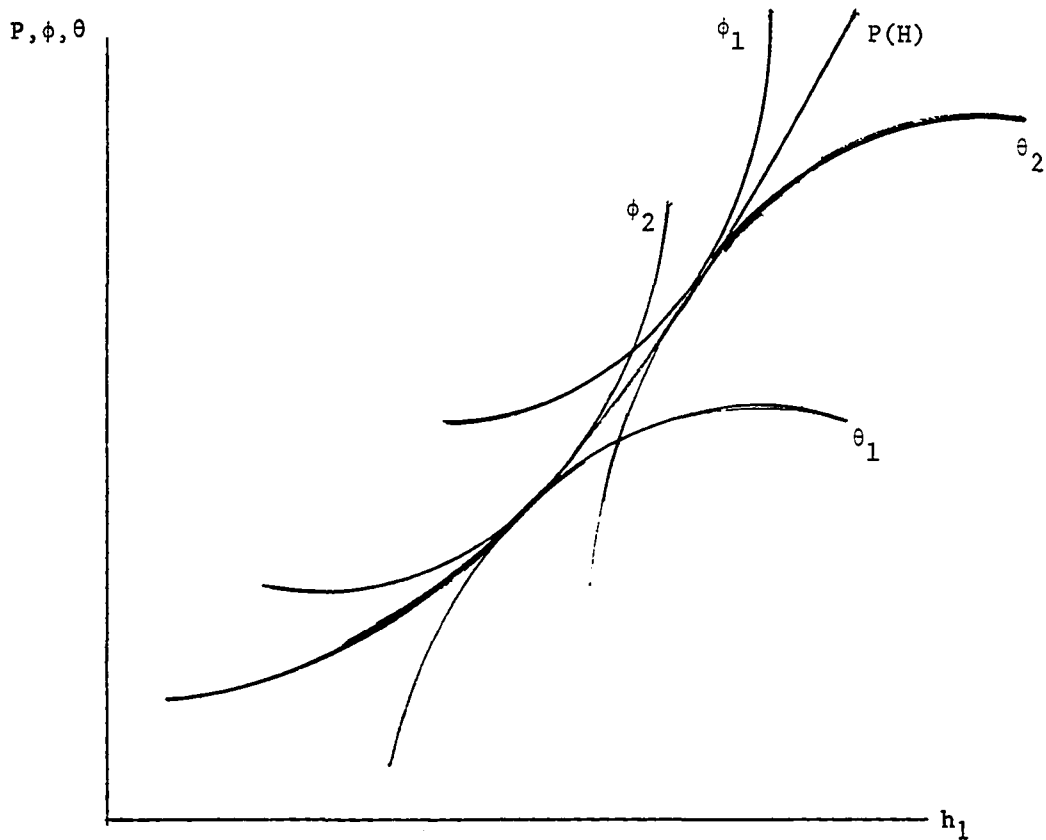


Figure 2.3. The hedonic price function represented at the joint envelope of a family of offer functions and a family of value functions

As seen from Rosen's description of the derivation of the implicit price function, a basic assumption of hedonic models is that the housing market is in equilibrium. There is an issue of whether the market clearing implicit prices obtained from the hedonic price function depict a short-run or a long-run equilibrium. If the available quantity of housing is fixed, then consumers may only bid on the available housing

stock. In this case only a short-run equilibrium is attainable.

Harrison and Rubinfeld made this assumption when estimating the demand curve for air quality. They assumed that the supply of air quality is perfectly inelastic with respect to price at each residential location.

If the supply of housing is endogenous, then a long-run equilibrium is attainable. Nelson assumes that the supply of air quality is price responsive. The interpretation of the exogeneity or endogeneity of the housing characteristic is a question which must be answered if the implicit prices are to be used to estimate a demand curve for an individual characteristic. If the supply of the characteristic is exogenous, then the implicit prices paid by consumers at varying levels of the characteristic may be regressed against the quantities demanded, income, and other household variables that influence tastes and preferences, to obtain the fully identified inverse demand function (Freeman, 1979b). If, however, the supply of the characteristic is endogenous then both a demand and supply curve must be estimated. Freeman (1979b) pointed out that the issue of whether supply is exogenous or endogenous depends on the speed at which the supply side adjusts to price changes relative to the speed at which housing prices adjust to changes in supply.

In this study, no attempt to estimate the demand curve for energy efficiency is made. It is hypothesized, however, that the supply of energy efficient homes is price responsive. Increasing insulation, caulking, retro-fitting with solar and purchasing high energy efficiency

furnaces are all possible ways in which the current housing stock may be changed as the implicit price of energy efficiency increases.

In this chapter, the theoretical justification for the use of the cost minimizing linear programming model has been discussed. The underlying theory of the hedonic model has also been examined and the assumptions implicit in the use of this model have been discussed. The next chapter will focus on the actual formation of the hedonic model. The empirical development of the hedonic technique will be discussed and the relevant issues will be explored. The actual data, model estimation, and results of the hedonic model constructed in this study will be examined.

CHAPTER 3. HEDONIC PRICE MODEL

In the previous chapter, it was demonstrated that the hedonic price function represents a joint envelope of a family of consumer value functions and a family of producer supply functions. Empirically, the process of obtaining the appropriate hedonic price function is accomplished by regressing house prices, site values, or rents against the price determining structural, neighborhood and environmental characteristics. The implicit prices associated with the individual characteristics may be obtained by taking the first derivative of the hedonic price function with respect to each characteristic. Many empirical studies of this sort have been undertaken. Much variation exists among the studies regarding the relevant dependent and independent variables to include in the model, the form of the functional relationship between the price of the house and its vector of price determining characteristics, and the size of the relevant housing market that the model represents.

In the first section of this chapter, a selection of studies from the hedonic literature is discussed, emphasizing the above issues. The objective of the hedonic price model in this study is examined in the second section. The third section describes the data sample from Des Moines, Iowa which was used in this study. The estimation procedure is described in the fourth section, and the final section will discuss the model results.

Discussion of Selected Articles from the Hedonic Literature

An early application of the hedonic method was made by Griliches in 1971. He regressed the prices of automobiles against their characteristics, in order to obtain the implicit price associated with each individual characteristic. These implicit prices were then used in the construction of a price index. The process Griliches followed was to "Derive implicit specification (quality) prices from cross-sectional data on the price of various 'models' of the particular item and use these in pricing the time series changes in specifications of the chosen (average or representative) item" (Griliches, 1971a, p. 56).

Since Griliches' application of the hedonic technique, numerous other hedonic studies have been made, particularly of the housing market. There are several issues which must be considered when applying the hedonic technique. The housing market to which the hedonic technique is to be applied must be defined. Is the housing market a national market in which suppliers and demanders have the geographic mobility to arbitrage all differences in the hedonic price function across geographic locations? Do the boundaries of a city or Standard Metropolitan Statistical Area (SMSA) denote the boundaries of a housing market? Or, is the market segmented even further according to the race or income of the buyers, the age or price range of the homes, or other possible subdivisions?

The possible existence of a national housing market was explored by Linneman (1980). If suppliers and demanders in the housing market are

geographically mobile enough to arbitrage the hedonic price function across location, then the relevant sample to use in a hedonic study of the housing market is a national sample of housing units. Linneman asserted that if the national housing market hypothesis is correct then "the use of local samples to estimate the hedonic price functions will induce sample selection bias to the extent that the local sample is not a random sample of the national sample" (p. 57).

To test this hypothesis, Linneman obtained maximum likelihood estimates of housing characteristic coefficients for a sample of observations from the 34 largest cities in the U.S. He then compared the price estimates obtained by using the national sample with price estimates obtained by using a sample from the individual cities of Chicago and Los Angeles. Linneman examined the owner and renter markets individually and found that one-third of the coefficients in the implicit price function of owner-occupied homes in the Chicago area, and 14 percent of the coefficients in the implicit price function of rented homes in Chicago, were more than 1.9 standard errors different from the full sample estimates. For Los Angeles, these percentages were 24 and 27, respectively. Based on these results, Linneman stated that the evidence of a national housing market is not conclusive, but relevant enough to warrant further investigation.

Several researchers have considered the possibility that a given city does not compose a single housing market, but that submarkets exist within SMSAs, and that a separate implicit price function should be

obtained for each of the individual submarkets. Straszheim (1974) first addressed the issue of market segmentation. He stated that "the central problem in estimating hedonic equations involves the delineation of homogeneous submarkets" (p. 404).

Conflicting evidence exists concerning the need for market segmentation. In a study on the effect of air pollution on property values, Nelson (1978) stratified his sample of Washington, D.C. homes into urban and suburban categories. He did not find conclusive evidence that the hedonic price functions for the two subcategories were different. Schnare and Struyk (1976) tested for market segmentation in the Boston housing market of single-family owner-occupied homes. They identified potential submarkets and then estimated a hedonic price function for each of the submarkets and for the sample as a whole. They then tested for significant differences between the parameters of the submarkets and those of the market as a whole. Their analysis indicated that the overall effect of the difference between the hedonic price function which was fit to the entire sample, and the hedonic price functions which were fit to the individual submarkets, was small. Significant differences were found in some of the individual parameter estimates, however.

Freeman (1979a) addressed the issue of market segmentation and stated that two conditions are necessary to have different hedonic price functions existing in an urban area, i.e., to have submarkets. "First purchasers in one market stratum must not participate significantly in

other market strata." And, "The second condition is that either the structure of demand, the structure of supply, or both must be different across regions" (p. 163).

For purchasers in one market strata not to participate in other market strata there must be some barrier preventing their participation. Possible barriers which Freeman cited are: barriers due to geography, discrimination, lack of information, or a desire for ethnically homogeneous neighborhoods. In the housing market used in this study, the SMSA of Des Moines, Iowa, the possibility of such market barriers seems less likely than in the Washington area examined by Nelson, or the Boston area examined by Schnare and Styruick. Des Moines does not have any major geographic barriers which cause market segmentation. In addition, Des Moines is much smaller than the Boston or Washington metropolitan areas, making it less likely that individuals would be forced to locate in a particular segment because of their work location. Finally, there are not any clearly defined homogeneous ethnic areas, nor does racial discrimination appear to be a significant problem in Des Moines.

Once the housing market to be used in the study is defined, the dependent and independent variables which are to be included in the hedonic price function must be identified. The dependent variable may be either pure land rent (the value of the site) or the price of housing.¹ Since energy efficiency is a characteristic of the house

¹The form of price of housing used in hedonic models may vary. For renter occupied housing, annual or monthly rent may be used. For owner occupied housing, owner assessed values, market prices, or a form of gross rent may be used.

itself (and the heating system included within it), the price of housing is used as the dependent variable in this study, rather than site value.

Previous hedonic studies exhibit a wide variation in the independent variables which they include in the implicit price function. Many of the studies are based on census tract level data, therefore, their unit of observation is median census tract levels (see Nelson (1978), Harrison and Rubinfeld (1978), Bloomquist and Worley (1981), and Halvorsen and Pollakowski (1981)). These studies are limited in the choice of structural characteristics which they may include as independent variables to those characteristics included in the Current Housing Reports compiled by the Bureau of the Census. For each tract, the census contains data on the number of housing units:

- lacking plumbing,
- having own kitchen,
- by number of rooms,
- by year built,
- by form of heat,
- with basement,
- with more than one bathroom, and
- with air conditioning.

Studies which use individual houses as their unit of observation (see Kain and Quigley (1970), Linneman (1980), Johnson (1981), and Schnare and Struyk (1976)) have a much wider selection of structural characteristics which may be included as independent variables. These

studies have used both individual interviews and multiple listing services as sources of data on the structural characteristics of each home. These studies typically include the type of information found in census tract studies, as well as information on the floor area of each home, lot size, kitchen appliances, number of fireplaces, and a variety of other characteristics.

There is an equal lack of consensus in the literature concerning the neighborhood characteristics which should be included in the hedonic price function. Among some of the more frequently included neighborhood variables are: distance to the central business district (CBD), percentage of blacks in the neighborhood, median schooling of adults or median income of residents, school quality, and crime rate. Studies examining the effect of air quality on housing or site value include one or more pollution variables in the hedonic price equation, along with the other neighborhood variables.

The literature offers little guidance on which structural and neighborhood characteristics to include in the hedonic price function. Freeman (1979a) pointed out that only exogenous characteristics should be included as right hand side variables. Endogenous characteristics, such as household income, should be excluded from the list of explanatory variables.¹ Butler (1982) stated that "In principle, all characteristics relevant to the determination of the market price...should be included"

¹Freeman does state, however, that the median income level of each census tract may be included to reflect the socioeconomic status of the neighborhood.

(p. 97). In reality, however, this is not possible. No data are available on many of the price determining characteristics, and as Butler pointed out, "Even without data constraints, the intrinsic clustering of characteristic combinations into a relatively small number of configurations leads to considerable multicollinearity in estimates employing a generous selection of the relevant variables" (p. 97).

Concluding that all hedonic price models must be misspecified, in that the complete list of price determining variables may not be included, Butler attempted to estimate the impact of this misspecification. He estimated two hedonic models of owner-occupied housing in St. Louis: a restricted model containing only four explanatory variables, and a more extensive model containing ten explanatory variables. Butler found that the changes in the standard error of the estimate and \bar{R}^2 caused by excluding six of the explanatory variables were only 4.7 and 7.9 percent, respectively. The coefficient bias in the restricted model (due to the excluded variables) was substantial for only one of the explanatory variables. This variable had a .55 correlation with one of the excluded variables.

Based on these findings Butler concluded that "the practical impact of these biases (due to excluded explanatory variables) is small" (p. 106). He noted, however, that these results apply only to the bias of structural characteristics and that the apparent insensitivity of structural coefficients to changes in the included variables does not necessarily carry over to neighborhood characteristics. Butler stated that "an attempt to determine the effect of race on rents by estimating

an equation containing only structure variables and a measure of neighborhood racial composition would in all likelihood yield a seriously biased coefficient for the latter variable" (p. 107).

While Butler examined the ramifications of excluded variables in the hedonic price function, Griliches explored the hazard of including too many explanatory variables. Griliches (1977) pointed out that an excessive number of independent variables in the hedonic equation can result in a serious downward bias in the estimated coefficients (p. 12). This is particularly true, he argued, when the variable of key interest is subject to measurement error. Griliches demonstrated this with a hedonic model designed to estimate the effect of schooling on earnings. The measure of schooling used in the model is subject to a measurement error. Griliches found that as additional independent variables (which are correlated with schooling) were added to the hedonic equation, the estimated coefficient on schooling approached zero and the measurement errors were magnified.

Neither Butler nor Griliches offer a systematic method for determining what explanatory variables to include in the hedonic price function; however, they illustrate the issues which must be considered. Butler's findings indicate that the danger of coefficient bias due to excluded variables is greater for the coefficients of neighborhood characteristics than for the coefficients of structural characteristics. This is because nonstructural characteristics "are typically more highly correlated" (p. 107). Butler's results also reveal that the coefficient bias of structural coefficients due to excluded variables is usually

small; however, it can be significant if the the included variable is highly correlated with the excluded variable.

Griliche's findings reveal that one must be aware of the problems caused by including an excessive number of explanatory variables as well. An increase in the number of explanatory variables increases the probability of encountering multicollinearity problems. To the extent that the increased number of explanatory variables cause multicollinearity, the added variables will increase the uncertainty concerning the true coefficient estimates and will increase the difficulty of ascertaining the separate effects of the individual housing characteristics (Judge et al., 1982). Griliche's findings also reveal that the measurement error of a given characteristic is magnified as the number of included variables is increased. In determining the appropriate variables to include in the hedonic model, therefore, "we must continuously search for the passage between the Scylla of biased inferences due to left-out and confounded influences and the Charybdis of overzealously purging our data of most of their identifying variance, being left largely with noise and error in our hands" (Griliches, 1977, p. 13).

A fundamental problem with hedonic models is that there does not exist any a priori theoretical foundation for choosing among alternative functional forms of the implicit price function. As stated in the discussion of the theoretical foundation of the hedonic technique, the implicit price function represents a joint envelope of a family of consumer value functions and a family of supplier offer functions (Rosen,

1974). The implicit price function itself reveals nothing about the underlying offer and value functions which generate it, with the exception of two special cases: (1) if there is no variance in cost factors or technology among firms and all firms are identical, then the family of offer functions degenerates into a single surface and the implicit price function represents the unique offer function; and (2) if buyers are identical then the family of value functions collapses into a single value function which is represented by the implicit price function (Rosen, 1974). Barring these two special cases, the form of the implicit price function may not be determined a priori from assumptions about the underlying supply or demand conditions.

A third special case exists when the characteristics of the compound good are completely divisible. There is no a priori reason to expect the implicit price function to be linear; however, Rosen demonstrated that if the characteristics of the compound good are fully divisible then the nonlinear portions of the implicit price function may be ruled out as uneconomical. To demonstrate this, assume that the price of the good can be written as a function of the goods characteristics:

$P(G) = p(g_1, g_2, \dots, g_n)$; and that $g_1 = \frac{1}{t}g_2$ and $P(g_1) < \frac{1}{t}P(g_2)$, where t is a scalar and $t > 1$. In this case, t units of characteristic g_1 could be purchased in place of g_2 , and transactions in the convex portion of $P(G)$ would be ruled out. Further, suppose $g_1 < g_2 < g_3$ and $P(g_2) > \delta P(g_1) + (1-\delta)P(g_3)$, $0 < \delta < 1$ and $g_2 = g_1 + (1-\delta)g_3$. Here, the buyer may obtain the satisfaction associated with characteristic g_2 more economically by purchasing a linear combination of g_1 and g_3 rather than

by purchasing g_2 itself. In this manner, the concave portions of $P(G)$ would be ruled out as uneconomical (Rosen, 1974).

When the characteristics of a composite good are not fully divisible, however, arbitrage activities such as the ones described above are not possible and it may not be expected that the implicit price function is linear. Such is the case with a house; each house is associated with a given set of characteristics and these characteristics may not be rearranged with the characteristics of other houses on the market. Due to the fact that nonlinearity may not be ruled out, it is important not to place too many restrictions on the implicit price function initially, and to test alternative functional forms.

A criticism of many hedonic models is that functional form is chosen on the basis of convenience. Linear, semi-log, and log-linear forms are frequently used in hedonic studies because of their ease in estimation. An example of a hedonic study using both a linear and semi-log price function is one by Kain and Quigley (1970). Their study was designed to measure the value of housing quality in the city of St. Louis. The data were obtained by three separate surveys of approximately 1,500 households and dwelling units in the summer of 1967. These surveys provided extensive information on 39 quality variables of the sample homes. These 39 quality variables were then aggregated by factor analysis into five factors which accounted for 60 percent of the variance among the 39 original quality variables. Separate hedonic price functions were fit to the rental and owner-occupied homes in the survey. A linear price

function was used for the rental homes and a semi-log form was used for the owner-occupied homes.

Schnare and Struyk (1976), estimated the implicit price function for a sample of 2,195 single-family homes located in the suburbs of Boston, using both a linear and semi-log form of the hedonic price function. Other examples of studies using linear and/or semi-log functional forms are: Johnson (1981), Nelson (1978), and Dale-Johnson (1982).

Harrison and Rubinfeld (1978) used a mixture of linear, log, and squared specifications of variables to find a functional fit. Their study is based on 1970 data from census tracts in the Boston Standard Metropolitan Area (SMSA). A hedonic model was constructed to measure the impact of air quality on housing prices. The dependent variable used is the median value (MV) of owner-occupied homes in each census tract. The independent variable used to indicate air quality is the concentration of nitrogen oxides (NOX) in each census tract. To estimate a nonlinear term in NOX, NOX^p was included in the equation. It was found that the best statistical fit was obtained when $p = 2$, and the dependent variable was entered in log form.

Box and Cox (1964) suggested a methodology which could be used to find the appropriate functional form in hedonic models. The Box-Cox model can be written as:

$$\frac{y_0^{\lambda_0 - 1}}{\lambda_0} = \beta_0 + \sum \beta_i \frac{x_i^{\lambda_i - 1}}{\lambda_i} + \varepsilon. \quad (3.1)$$

Several hedonic studies have utilized the Box-Cox method, however, in most cases severely limiting assumptions have been placed on the form that the model may take due to the fact that an unrestricted model is complex and costly. Three examples of studies using a restricted Box-Cox method are articles by Goodman (1978), Linneman (1980), and Bloomquist and Worley (1981).

Goodman (1980) employed the Box-Cox methodology in determining the form of the hedonic price function for a sample of single-family homes sold in the New Haven SMSA between 1967 and 1969. He broke the data down into 15 submarkets and assumed that there was a single best functional form for the entire metropolitan area. Since the 15 submarkets were independent, the joint maximum likelihood function for the SMSA was the product of the individual likelihood functions of each of the submarkets. In determining the functional form Goodman restricted the model so that all λ_i were set equal to one, therefore, only the optimal value of λ_0 was searched for. The value $\lambda_0 = 0.6$ was found to maximize the joint likelihood function; therefore, both the linear and semi-log forms of the model were rejected.

Linneman (1980) constructed a hedonic model of the Chicago housing market, the Los Angeles housing market, and a national housing market. Linneman followed the same procedure as Goodman and restricted the five continuous independent variables in his model to the linear ($\lambda_i = 1$) and log-linear ($\lambda_i = 0$) forms. Furthermore, he assumed that the same power transformation was appropriate for all the independent variables. He made these restrictions due to limited computer funds and because

"preliminary investigation indicated that the value of the likelihood function was substantially more sensitive to changes in the specification of the dependent variable than to changes in the specification of the independent variables" (p. 53). Linneman determined that a natural logarithmic transformation of the dependent variable (property value) and a linear transformation of the five continuous independent variables provided the best statistical fit. He noted, however, that it could not be rejected (at the 95 percent level) that the true λ_0 is between 0.2 and -0.2 when $\lambda_1 = 1$ and between 0.3 and -0.3 when $\lambda_1 = 0$.

Bloomquist and Worley (1981) constructed a hedonic price function of owner-occupied housing in Springfield, Illinois using block and block group data from the 1970 Census. They utilized the Box-Cox method but restricted the search to forms where the power transformation of all variables is the same. They found that the 0.1 power transformation maximized the likelihood function and that "0.1 is significantly different from the linear form 1.0, and 0.1 is not significantly different from the natural logarithmic form 0.0" (p. 216).

In summary, all three of the Box-Cox studies cited here rejected the linear model which is frequently used in hedonic studies, and the Goodman study also rejected the semi-log model.

A study by Witte, Sumka, and Erekson (1979) utilized a quadratic model in an attempt to allow for nonlinearity. The model they used is:

$$R = \alpha + \sum_{i=1}^5 \beta_i X_i + \sum_{i=1}^5 \sum_{j=1}^5 \delta_{ij} X_i X_j + \sum_{i=1}^2 \gamma_i D_i + U_i \quad (3.2)$$

where R is annual contract rent; the α , β_i 's, δ_{ij} 's, and γ_{ij} 's are parameters to be estimated; the x_i (x_j) are the five continuous independent variables; the D_i 's represent two dummy variables which were included in the model; and U_i is a normally distributed stochastic error term.

An alternative form for hedonic models was suggested by Halvorsen and Pollakowski (1981). They suggested a functional form for hedonic price equations "that combines the best features of the Box-Cox and flexible form approaches" (p. 38). It is a general functional form which incorporates all other functional forms of interest as special cases. This general functional form, which they call a quadratic Box-Cox functional form, is:

$$P^\theta = \alpha_0 + \sum_{i=1}^m \alpha_i Z_i^\lambda + \frac{1}{2} \sum_{i=1}^m \sum_{j=1}^m \gamma_{ij} Z_i^\lambda Z_j^\lambda \quad (3.3)$$

where P is price, the Z_i are attributes, $\gamma_{ij} = \gamma_{ji}$ and P^θ and Z_i^λ are Box-Cox transformations,

$$P^\theta = \frac{P^\theta - 1}{\theta}, \quad \theta \neq 0,$$

$$= \ln P, \quad \theta = 0,$$

$$Z_i^\lambda = \frac{Z_i^{\lambda-1}}{\lambda}, \quad \lambda \neq 0,$$

$$= \ln Z_i, \quad \lambda = 0.$$

The model used by Witte et al. (1979) (described above) is a special case of this model in which the restriction $\theta = \lambda = 1$ is imposed. The translog form, generalized square root quadratic form, square root quadratic form, generalized nonhomogeneous version of the generalized Leontief form, and the frequently used semilog form may all be obtained from the quadratic Box-Cox functional form by imposing appropriate restrictions. In order to test which particular functional form is appropriate, the restrictions corresponding to that functional form are tested using a likelihood ratio test.

Halvorsen and Pollakowski applied this model to a sample of 5,727 single-family owner-occupied dwelling units in the San Francisco Bay Area. They estimated alternative forms of the hedonic price equation using ordinary least squares. They also estimated the unrestricted quadratic Box-Cox form by performing a grid search over values of θ and λ between -1.0 and 2.0 and determining which of these values maximize the log likelihood function. The optimal values obtained for θ and λ by this method were 0.06 and 0.28, respectively. A 99 percent confidence region was constructed around $\theta = 0.06$ and $\lambda = 0.28$ and the shape of this confidence region indicated that "the value of the log likelihood function was substantially more sensitive to the value of the

transformation parameter for the dependent variable than to the value of the transformation parameter for the independent variables" (p. 45).

Freeman (1979b) pointed out that there are economic implications associated with alternative functional forms. To highlight this point he defined a property value equation:

$$R_i = R(P_i, \dots) \quad (3.4)$$

where: R_i = annual rent of the i^{th} location and

P_i = air pollution at the i^{th} location.

He then specified eight alternative forms of the hedonic price function: linear, quadratic, log, semi-log, inverse semi-log, exponential, semi-log exponential, and Box-Cox transformation. Freeman demonstrated that each of these functional forms has implications about the relationship between the marginal implicit price of pollution and both the level of other housing attributes and the level of pollution itself. Of the eight functional forms listed above, only two, the log and the Box-Cox transformation, allow the implicit price of pollution to depend on the level of other characteristics. All of the other models impose independence between each implicit price and the levels of other housing attributes.

The second derivative of the hedonic price function determines the relationship between the marginal implicit price of an attribute and the level of that attribute. Consider an attribute, such as energy efficiency, which has a positive valuation and therefore a positive

implicit price. Of the eight functional forms discussed by Freeman only one, the linear form, imposes independence between the implicit price of energy efficiency and the level of energy efficiency itself. The log, semi-log, and inverse semi-log forms all have positive second derivatives, indicating that the implicit price function has the shape shown in Figure 3.1, Panel a. Use of these forms implies that the marginal implicit price of energy efficiency increases as the efficiency level increases. The quadratic, exponential, semi-log exponential, and Box-Cox transformation all allow the second derivative to be either positive or negative. This indicates that these models will allow the marginal implicit price of energy efficiency to be either positively or negatively related to the level of energy efficiency, i.e., the implicit price function could have the form shown in either Panel a or b.

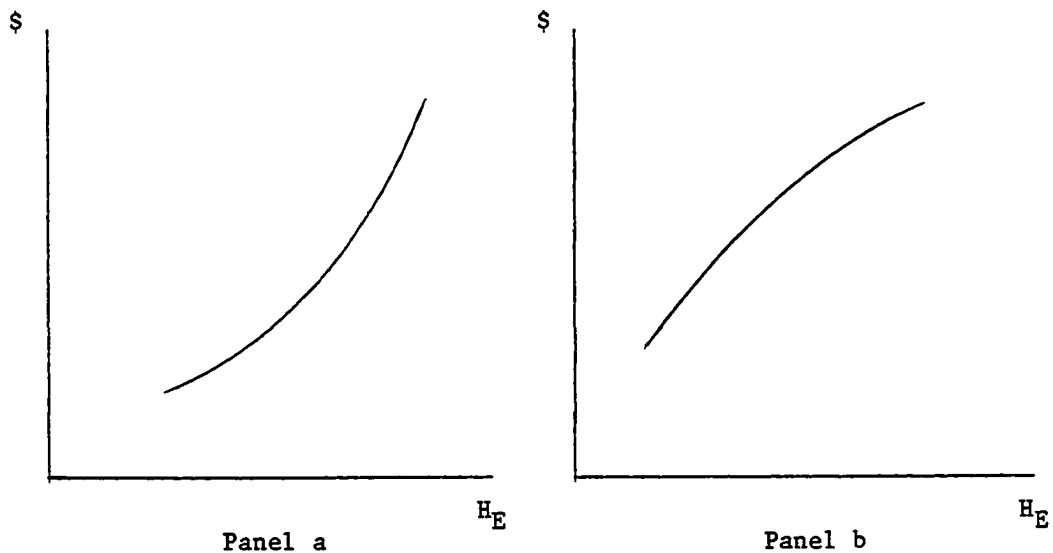


Figure 3.1. Alternative forms of the implicit price function

The results of previous studies indicate the importance of using a flexible functional form to estimate the hedonic price function. Authors who have experimented with flexible functional forms (Linneman (1980), Goodman (1978), and Halvorsen and Pollakowski (1981)) found that their fit was significantly better than the fit of the linear, semi-log, and log-linear models frequently used in hedonic studies.

Additionally, Rosen's study reveals that there is no a priori reason to choose a linear, semi-log, log-linear, or any other functional form. He demonstrated that the hedonic price function is a reduced form equation and that its functional form cannot be determined a priori from the form of the underlying supply and demand equations.¹ The form of the hedonic price function can be assumed to be linear in the special case in which the good characteristics are fully divisible; however, this is not the case for the characteristics of a house.

Since the form of the hedonic price function cannot be deduced from the underlying supply and demand equations, the economic relationship between the implicit price of a given characteristic and the level of that characteristic or other characteristics may not be determined a priori. It is important, therefore, to choose a functional form which does not predetermine the form of these economic relationships. Freeman's findings indicate that out of the eight functional forms he examined,² the Box-Cox model allows for the most flexibility in the

¹Two exceptions which Rosen (1974) points out are when all firms are identical or when all buyers are identical.

²Freeman (1979b) examined the economic implications of the linear, quadratic, log-linear, semi-log, inverse semi-log, exponential, semi-log exponential, and Box-Cox functional forms.

form of the resulting economic relationships. Freeman's study reveals that the Box-Cox model is the only one of the functional forms considered which both: (1) allows the implicit price of a given housing characteristic to depend on the level of other characteristics and (2) allows the implicit price of the given characteristic to either increase or decrease as the level of the characteristic itself increases.

In this section, the relevant issues in the construction of a hedonic price model were discussed. The issue of relevant market size was examined. The question of what variables should be included in the hedonic price function was discussed. The types of variables used in previous models were stated and the danger of coefficient bias due to excluded variables, as well as the estimation problems due to an excessive number of included variables, were examined. Finally, the issue of choice of functional form was discussed. The examination of previous studies revealed the importance of using a flexible form to estimate the hedonic price function. The following section will discuss the objective of the hedonic price model which is constructed in this study.

Objective of the Hedonic Price Model

As discussed in the introduction, a key question addressed in this study is: Does the cost minimizing fuel saving investment mix change as the period of time the home is to be owned is altered? The answer to this question, of course, depends on the resale value of each fuel saving

investment. For example, if an individual planned to own a newly constructed home for only one year, it would not be cost effective for him to install any fuel saving investment having a pay back period of more than one year unless he would receive a premium for the house due to the inclusion of the fuel saving investment.

The objective of the hedonic price model is to estimate the impact of energy efficiency on housing prices. This information is then used to estimate the resale value of fuel saving investments. As demonstrated in the previous section the hedonic technique is a method of deriving the implicit prices of good characteristics. In this study, the hedonic technique is applied to a sample of homes from Des Moines, Iowa. The form of the hedonic price function relating housing prices to housing characteristics in Des Moines is estimated (equation 2.11), and the implicit price of each individual characteristic is obtained by differentiating the hedonic price function with respect to that characteristic, holding all other factors constant (equation 2.12).

Ideally, each fuel saving investment could be included in the vector of housing characteristics, $H = (h_1, h_2, \dots, h_n)$. The implicit price of each fuel saving investment could then be obtained directly by the hedonic technique. This is not possible, however, for two reasons: (1) no information on the fuel saving investments present in the sample homes is available and (2) even if such information was available, a problem of multicollinearity may exist. For example, there may be a high correlation between homes having passive solar applications and high insulation levels. This would cause the variance of the coefficients on

these two characteristics to be large, making the coefficient estimates unreliable. Also, a high degree of correlation would make it difficult to determine the separate effects of insulation and passive solar applications on housing prices (Judge et al., 1982).

In light of these two problems, an indirect method of estimating resale values is chosen. First, a measure of the heating efficiency of each home is obtained. Next, the hedonic technique is used to estimate the "implicit price" of energy efficiency, i.e., the change in the selling price of a house due to an increase in efficiency, *ceteris paribus*. Finally, given the implicit price of heating efficiency and information concerning the increase in efficiency brought about by each fuel saving investment, the resale value of each fuel saving investment may be obtained.

Data Section

The sample used in this study consisted of 234 homes sold in Des Moines, Iowa, during the period January 1982 through June 1982. To construct the hedonic model, information on the selling price, structural, and neighborhood characteristics of each home in the study was needed. Information on the selling price and several descriptive characteristics of each home were obtained from the Greater Des Moines Board of Realtors. Information on the age and square footage of floor area was obtained from the city and county assessors offices of Des Moines. For each home, the median income of the appropriate census tract was obtained from the 1980 census, and the distance of the home

from the central business district was measured. Table 3.1 lists the available data. The definition of each variable is given, along with its source. Table 3.2 indicates the mean value and standard deviation of each variable.

A key independent variable in the hedonic model is, F , the adjusted fuel bills per square-foot of floor space for each home in an average heating season.¹ The adjustment accounts for differences in internal temperature settings among the sample homes. The variable F was constructed to represent the relative energy efficiency of each house in the sample, i.e., the lower the level of adjusted fuel bills per square-foot, the higher the level of energy efficiency. F was calculated by determining the fuel expenditures per degree-day per square-foot for each home and then multiplying by the average number of degree-days in a Des Moines heating season:

$$F_i = (\$_i / (HDD_i)) (1 / Area_i) * 6,550 \quad (3.5)$$

where: F_i = adjusted fuel expenditures per heated square-foot of house

i. F_i reflects the heating expenditures per square-foot house i would incur in an average heating season if the

¹To calculate F , information on the December 1982 through February 1983 fuel expenditures of each home was obtained from Iowa Power, information on the internal temperature setting of each home was obtained from the homeowners, and information on the number of degree-days in the 1982 heating season was obtained from the Energy Extension Office at Iowa State University. An average heating season was represented by the average number of degree-days in an Iowa heating season (6,550).

Table 3.1. Observed housing characteristics for Des Moines sample

<u>Housing characteristic</u>	<u>Source^a</u>
PRICE: Selling price of house	1
BED: Number of bedrooms	1
BATH: Number of bathrooms	1
FAMRM: Family room present ^b	1
DINRM: Dining room present ^b	1
SQFT: Square-feet of floor area	2
LOT: Square-feet of lot area	1
BASE: Basement present ^b	1
DISH: Dishwasher present ^b	1
RANGE: Cooking range present ^b	1
DISP: Garbage disposal present ^b	1
CA: Central air-conditioning present ^b	1
WA: Window air-conditioner present ^b	1
GAR: Garage present ^b	1
GAR1: Single garage present ^b	1
GAR2: Double garage present ^b	1
FP: Fireplace present ^b	1
AGE: Age of house	2
F: Adjusted fuel bills per square-foot of heated floor area	3
F*: Predicted fuel bills per square-foot of heated floor area	4
NBHD: Median income of appropriate census tract, used as a proxy for neighborhood status	5
LOC: Miles from the central business district	6

- ^aSources: 1--Multiple Listing Service, Des Moines, Iowa.
 2--City and County Assessor's Office.
 3--Based on information from homeowners, Iowa Power, and the Energy Extension Office at Iowa State University.
 4--Estimated in this study.
 5--1980 Census of Population and Housing Census Tracts, Des Moines, Iowa, Standard Metropolitan Area.
 6--Des Moines city map.

^bIndicates a qualitative variable.

Table 3.2. Observed housing characteristics for Des Moines sample

<u>Variable name</u>	<u>Mean</u>	<u>Standard deviation</u>
PRICE	67,082	44,197
BED	3.0256	.79658
BATH	1.6207	.66939
FAMRM	.64957	.47813
DINRM	.63248	.48316
SQFT	1,253.0	530.87
LOT	11,599	9,868.7
BASE	.92308	.26704
DISH	.54701	.49885
RANGE	.74359	.43759
DISP	.68376	.46600
CA	.66667	.47242
WA	.17521	.38096
GAR	.89316	.32314
GAR1	.35897	.48073
GAR2	.53419	.49990
FP	.39744	.49042
AGE	29.543	22.066
F	.43809	.14906
F [*]	.34819	.09677
NBHD	21,079	5467.1
LOC	4.7223	3.3939

internal temperature was maintained at 65° F.

HDD_i = heating degree-days for household i in the December 1982 through February 1983 billing period.

$Area_i$ = square-feet of heated floor area of house i .

6,550 = the average number of heating degree-days in a Des Moines heating season (using 65° F as the base temperature).

A heating degree-day is a one degree difference between the internal temperature of the home and the external temperature over a 24 hour period. Therefore, the value of HDD_i varies among homes according to the billing period and the internal temperature of the home:

$$HDD_i = BDD_i + D_i(T_i - 65^\circ) \quad (3.6)$$

where: BDD_i = heating degree-days in the December 1982 through February 1983 billing period of house i (using 65° F as the base temperature),

D_i = number of days in the billing period for house i , and

T_i = internal temperature of house i .

In initial regressions of the hedonic price model, F was included as an independent variable. The coefficient on F in these initial regressions was positive. A positive coefficient indicates that, for a given home, an increase in the value of adjusted fuel bills per square-foot increases the expected selling price of the house, *ceteris paribus*. If the adjusted fuel bills per square-foot of each home were a true measure of the efficiency of the home, one would not expect an increase

in F (i.e., a decrease in heating efficiency) to cause an increase in the value of the home.

A positive coefficient on F may indicate that F is not a true measure of the efficiency of the sample homes. The adjusted fuel bills per square-foot of floor area of house i , F_i , may reflect the lifestyle of the occupants, as well as the structural efficiency of the house i . Although F was adjusted for the internal temperature setting of each home, it is possible that this adjustment was not complete due to imperfect information. An increase in F_i may, therefore, reflect a warmer internal temperature in the home, rather than a decrease in the structural efficiency of house i . Additionally, no adjustment was made for the use of appliances in the home. In homes that are heated by natural gas,¹ a high level of appliance usage will increase the cost of heating the home. Although the appliances provide heat for the home, many of them are operated by electricity rather than natural gas. Due to the fact that electricity is a more expensive energy source than natural gas in the Des Moines area,² use of many appliances will increase the cost of heating the home. An increase in F , therefore, may reflect an increase in appliance usage, or an increase in the internal temperature level, rather than a decrease in the structural efficiency of the home.

¹Over 90 percent of homes in the Des Moines study area are heated by natural gas (Clark Bruebaker, Iowa Power, Des Moines, personal communication, 1984).

²Assuming a 75 percent efficient furnace, the price of natural gas is \$7.28/MBTU whereas the price of electricity is \$22.27/MBTU (Erv Roberts, Iowa Power, Des Moines, personal communication, 1984).

F_i is used as a proxy for the true measure of the structural efficiency of house i . There is, however, an error component in F_i , since F_i reflects more than just the structural efficiency of house i . F_i is not a fixed, exogenous measure of the efficiency of each home and is correlated with the error term in the hedonic model. The coefficient on F_i in the hedonic price function, therefore, is biased (Judge et al., 1982). To conceptualize this problem, let:

$$F_i = F_i^* + u_i \quad (3.7)$$

where: F_i = the actual bills per square-foot, and

F_i^* = the component of fuel bills per square-foot which is due to the structural efficiency of the house.

Ideally, F_i^* would be included in the hedonic price model. F_i^* would be a fixed exogenous right hand side variable, reflecting only the true structural efficiency of the home. Unfortunately, however, no measure of F_i^* is available. The available measure, F_i , includes F_i^* as well as an error factor, u_i . To understand the problems created by using F_i , as opposed to F_i^* , in the hedonic model, assume for a moment that a simplified hedonic price function is estimated using heating efficiency as the only explanatory variable:

$$P(H)_i = \beta_0 + \beta_1 F_i^* + v_i, \quad (3.8)$$

however, using the measure of bills per square-foot that is available, the model is:

$$P(H)_i = \beta_0 + \beta_1(F_i - u_i) + v_i, \quad (3.9)$$

$$= \beta_0 + \beta_1 F_i - \beta_1 u_i + v_i$$

$$= \beta_0 + \beta_1 F_i + e_i,$$

where: $e_i = v_i - \beta_1 u_i$.

Clearly, F_i is correlated with e_i since F_i is determined in part by u_i and e_i is a function of u_i . Even if $E(u_i) = E(v_i) = 0$, the covariance between F_i and e_i is:

$$E[(F_i - E[F_i])(e_i - E[e_i])] = E[u(v_i - u_i \beta_1)], \quad (3.10)$$

$$= -\beta_1 E(u_i^2),$$

$$\neq 0.$$

Because the explanatory variable, F , is not independent of the error term, the least squares estimator is not unbiased. The least squares estimator is:

$$f = (F'F)^{-1}F'P(H) \quad (3.11)$$

recalling that, $P(H)_i = \beta_0 + \beta_1 F_i + e_i$

$$f = \beta_1 + (F'F)^{-1}F'e. \quad (3.12)$$

The expected value of the estimator is:

$$E(f) = \beta_1 + E[(F'F)^{-1}F'e]. \quad (3.13)$$

Since, as was shown above, F and e are not independent of each other, the last term does not equal 0; therefore, the $E(f) \neq \beta_1$ and the estimator is biased.¹

In order to resolve this problem, a predicted variable, \hat{F}^* , is represented by a linear combination of observable independent explanatory variables. This set of "instrumental" variables explain the component of fuel expenditures attributable to the structural efficiency of the home:

$$\hat{F}^* = x_1\pi_1 + Z_2\pi_2 + \dots + Z_k\pi_k \quad (3.14)$$

where x_1 is a column vector of ones and π 's are additional unknown parameters. The simplified hedonic model may now be written as:

$$P(H) = \beta_0 + x_1\beta_1\pi_1 + Z_x\beta_2\pi_2 + \dots + Z_k\beta_1\pi_k + v \quad (3.15)$$

$$\text{or, } P(H) = Z\pi_F\beta_1 + v.$$

¹This discussion is based on Judge et al. (1982), pp. 277-278 and 532-534.

To estimate β_1 , the value of $Z\pi_F$ must first be obtained. A two-stage least squares procedure is used. First, F is regressed on the vector Z to obtain:

$$\pi_F = (Z'Z)^{-1}Z'F. \quad (3.16)$$

Next, $P(H)$ is regressed on $F^* (= Z\pi_F)$ to obtain the least square estimators denoted by β_1 , as:

$$\beta_1 = (F'Z(Z'Z)^{-1}Z'F)^{-1}F'Z(Z'Z)^{-1}Z'P(H). \quad (3.17)$$

By this method, an unbiased estimator of the true efficiency measure, F^* , may be obtained. The first step in resolving the errors-in-variable problem present in this study, therefore, was to determine the appropriate vector of instrumental variables that F could be regressed on to obtain the predicted value of $F^* = \hat{F}^*$. To determine the vector of structural and locational variables which explain F^* , the actual fuel bills per square-foot, F , were regressed against all independent variables in the hedonic model (i.e., the structural and locational characteristics of the house). The variables which were significant in this regression (at a 95 percent confidence level), plus a dummy variable to indicate if the house was more than one story, were used as the set of instrumental variables. Next, the actual values of F were regressed against this set of instrumental variables and a predicted value, \hat{F} , was obtained.

The results of this regression are shown in Table 3.3. \hat{F} is a function of only exogenous variables; therefore, \hat{F} would be uncorrelated with the error term in the hedonic model. The variable NBHD was shown to be significant in explaining F and was included in the regression to obtain \hat{F} . The variable NBHD is a proxy for the social status of the neighborhood. NBHD, therefore, is likely to reflect the lifestyle of the occupants rather than the structural efficiency of the house. A new predicted variable, \hat{F}^* , was calculated, where \hat{F}^* equals \hat{F} minus the impact of NBHD. \hat{F}^* is an exogenous variable which reflects only the structural efficiency component of fuel bills per square-foot. \hat{F}^* was then used in the hedonic model as a measure of the true efficiency of each house.

Table 3.3. Regression results of obtaining \hat{F}^a

Dependent variable: F = adjusted heating bills per square foot		
<u>Independent variables</u>	<u>Estimated coefficients</u>	<u>t-ratio</u>
SQFT ^b	-.0117	-2.03
SQFT ^{2b}	.00038	2.62
LOC	-.00743	-2.15
BASEMENT AREA ^b	-.022096	-8.17
NBHD ^c	.00422	2.25
STORY	-.001567	-0.08

$$a_R^2 = .35.$$

^bMeasured in hundreds of square-feet.

^cMeasured in thousands of dollars.

Model Estimation

As discussed in the theoretical section "the hedonic technique is a method for estimating the implicit price of the characteristics which differentiate closely related products in a product class." (Freeman, 1979b, p. 78). In this study, the hedonic technique is used to estimate the implicit prices of housing characteristics in the Des Moines housing market. The function relating housing prices to housing characteristics is defined as:

$$P(H) = P(h_1, h_2, \dots, h_n) \quad (3.18)$$

where: h_1, h_2, \dots, h_n is a complete list of the price determining characteristics of the home and

$P(H)$ = the selling price of the house.

In light of the findings of Rosen (1974), Freeman (1979b), Halvorsen and Pollakowski (1981), Goodman (1978), and Linneman (1980)¹, a Box-Cox model selected for estimating the hedonic price function. A full Box-Cox model has the following form:

$$P^\theta(H) = \alpha_0 + \sum \alpha_i h_i^{\lambda_i} + u \quad (3.19)$$

where: $P^\theta(H) = \frac{P(H)^\theta - 1}{\theta}, \quad \theta \neq 0,$

$= \ln P, \quad \theta = 0,$

¹A discussion of these studies may be found on pp. 45-51.

$$h_i^{\lambda} = \frac{h_i^{\lambda_i} - 1}{\lambda_i}, \quad \lambda_i \neq 0,$$

$$= \ln h_i, \quad \lambda_i = 0.$$

Due to computational limitations, using a full Box-Cox model, in which the dependent variable and each of the independent variables may take on a different power transformation factor, was not feasible.¹ In solving for the appropriate θ and λ_i 's, it was necessary to constrain all of the λ_i 's to be equal; therefore, all the continuous independent variables have the same power transformation factor, i.e., all $\lambda_i = \lambda$.

If the values of θ and λ are constrained to equal 1, then the model reduces to a linear form. If θ and λ are constrained to equal 0, the model reduces to a log-linear form. If the value of θ is set equal to 0 and the value of λ is set equal to 1, then a semi-log model results. Therefore, all of the functional forms that are commonly used in hedonic models, the linear, semi-log, and log-linear forms, are subcategories of the Box-Cox model used in this study.

The Box-Cox procedure allows alternative functional forms to be compared by using the log likelihood function as a measure of each models "fit". In this study, a procedure was used in which the power transformation factor, θ , of the dependent variable, price, was parameterized. The value of θ was increased by increments of 0.10 between the range of -1.0 and 2.0. For each value of θ , the value of λ _

¹In the econometric computer package used in this study (White, 1981), it is not possible to allow each independent variable to have a separate power transformation factor in any regression having more than two independent variables.

was found which maximized the log likelihood function, where all the continuous independent variables were transformed by the value λ (qualitative variables are not transformed). This process results in a series of θ , λ combinations which may be used to specify the model. By comparing the value of the log likelihood function for each of these θ , λ combinations, the model which provides the best fit may be obtained.¹

As discussed in the previous section, the selection of independent variables is a key issue in the construction of a hedonic price function. One must be aware of the danger of biased coefficients due to excluded variables, as well the problems of multicollinearity and magnified measurement error due to an excessive number of included variables. The parameter search procedure described above was used to estimate two Box-Cox models containing alternative specifications of independent variables. A full model was estimated using all available information on the structural and neighborhood characteristics of homes in the Des Moines sample. Table 3.1 lists these variables and indicates their sources. The value of the log likelihood function for this specification of independent variables is at a maximum when the Box-Cox transformation, θ , of the independent variable, price, is equal to -0.10, and the Box-Cox transformation, λ , of the independent variables is equal to 0.30. The implicit prices of housing characteristics obtained from this model specification (denoted \hat{P}_1) are indicated in Table 3.4. Ten of the 19 variables included in this model are significantly different from zero at a 95 percent confidence level.

¹This procedure was used by Halvorsen and Pollakowski (1981).

Table 3.4. Implicit prices obtained using alternative specifications of independent variables

Variable name	Full model	Restricted model	$ \hat{p}_1 - \hat{p}_2 $
	\hat{p}_1^a	\hat{p}_2^b	$ \hat{p}_1 $
F ^{*c}	-11.48 [*]	-11.63 [*]	.0131
BED	6,798 [*]	6,363 [*]	.0640
BATH	8,524 [*]	8,733 [*]	.0245
FAMRM	3,762	3,592	.0452
LOT ^c	52.00 [*]	53.94 [*]	.0373
DISH	7,026 [*]	8,088 [*]	.1511
CA	3,181	3,741	.1482
WA	60.81	-	-
GAR	-	11,243 [*]	-
GAR1	11,541 [*]	-	-
GAR2	10,413 [*]	-	-
FP	1,340	1,324	.0119
AGE	-177 [*]	-157 [*]	.1130
NBHD ^d	28.31	28.32	.0004
LOC	69.31	191	1.7557
SQFT	14.28 [*]	17.29 [*]	.2108
BASE	11,008 [*]	11,182 [*]	.0158
DINRM	4,589	-	-
RANGE	2,477	-	-
DISP	2,408	-	-

^aImplicit price obtained using Box-Cox model with: $\theta = -0.10$; $\lambda = 0.30$.

^bImplicit price obtained using Box-Cox model with: $\theta = -0.10$; $\lambda = 0.29$.

^cMeasured in hundreds of square feet.

^dMeasured in hundreds of dollars.

*Indicates implicit price is significant at a 95 percent confidence level.

An alternative model, using a restricted specification of independent variables, was also estimated. In this model, four of the seven independent structural variables which were not significant in the full model were eliminated. RANGE and DISP were excluded from the restricted regression, while DISH, which was significant in the full model, was included. Since no other appliances or kitchen characteristics are included in the restricted equation, DISH serves as a proxy variable, denoting a modern kitchen. In the full model, neither DINRM, nor FAMRM were significant. It was thought that a problem of multicollinearity may have prevented either of these variables from being significant. DINRM, therefore, was excluded from the restricted model to see if FAMRM became significant. Finally, the window air-conditioning variable, WA, was excluded from the restricted model, and GAR1 and GAR2 were condensed into one variable, GAR. Neither of the neighborhood variables (LOC and NBHD) were significant in the full model, however, these were not excluded from the restricted model since no other neighborhood variables were available.

The value of the log likelihood function for the restricted specification of independent variables is at a maximum when the Box-Cox transformation of price, θ , is equal to -0.10 and the Box-Cox transformation of the independent variables, λ , is equal to 0.29. The implicit prices of housing characteristics obtained from this model specification (denoted as \hat{P}_2) are indicated in Table 3.4.

As revealed in Table 3.4, the implicit prices of the housing characteristics contained in both the full and restricted model are not

substantially different in the two models¹. For eight of the 13 variables common to both models, the implicit price differs between the two models by less than ten percent. The implicit price of \hat{F}^* , the key variable of interest in this study, changed by less than two percent due to the use of the restricted model. The only exception to the general pattern of similar implicit prices in the two models is LOC, the variable indicating the distance of each home from the central business district. The implicit price of LOC is over twice as large in the restricted model as it is in the full model.

The implicit prices of housing characteristics in this study do not appear to vary significantly with changes in the included independent variables. Further investigation was undertaken to determine if the implicit prices of housing characteristics were sensitive to changes in the functional form of the hedonic price model. As previously discussed, a Box-Cox model was used to estimate the hedonic price function. This model allows for flexibility in the form the hedonic price function may take. For comparison purposes, the linear, semi-log, and log-linear models, which are frequently used in hedonic models, were estimated. The restricted specification of independent variables was used in estimating these models. Table 3.5 indicates the estimation results of the Box-Cox model and the three alternative models. Table 3.6 indicates the implicit

¹These results are consistent with those of Butler (1982). Butler estimated both a full and a restricted hedonic price function of the St. Louis housing market. In general, he found that the implicit prices of the housing characteristics common to both models were not substantially different.

Table 3.5. Regression results of alternative functional forms

Variable name	Box-Cox model	Semi-log model	Log-linear model	Linear model
\hat{P}^*	-.59930D-1 (-2.78)	-.69151 (-3.68)	-.81908D-1 (-2.39)	-25,097. (-1.30)
BED	.69259D-1 (4.43)	.77859D-1 (3.41)	.27331 (4.42)	3,524.1 (1.50)
BATH	.60918D-1 (3.80)	.13066 (3.96)	.24044 (4.35)	17,572. (5.17)
FAMRM	.17812D-1 (1.58)	.54798D-1 (1.62)	.62828D-1 (1.84)	2,146.1 (0.62)
LOT ^b	.26746D-3 (5.27)	.67564D-3 (4.10)	.50288D-1 (2.65)	147.50 (8.69)
DISH	.40106D-1 (3.40)	.12338 (3.50)	.10237 (2.82)	4,517.8 (1.24)
CA	.18054D-1 (1.64)	.53285D-1 (1.60)	.70815D-1 (2.12)	-1,262.8 (-0.37)
GAR	.55751D-1 (3.92)	.16487 (3.87)	.16174 (3.68)	4,425.2 (1.01)
FP	.65645D-2 (0.67)	.25785D-1 (0.88)	.12633D-1 90.42	-915.32 (-0.30)
AGE	-.77898D-3 (-3.05)	-.23920D-2 (-3.13)	-.57025D-1 (-3.87)	-125.22 (-1.59)
NBHD ^c	.62638D-3 (1.31)	.60460D-3 (1.88)	.65490D-1 (1.00)	31.687 (0.96)
LOC	.28386D-2 (0.45)	.35840D-2 (-0.66)	.22325D-2 (6.81)	-331.45 (-0.59)
SQFT	.13519D-1 (5.38)	.24437D-3 (5.65)	.28305D-3 (1.46)	26.96 (6.05)
BASE	.55452D-1 (3.20)	.13381 (2.48)	.46395D-1 (2.95)	4,051.7 (0.73)
	$R^2 = .83$	$R^2 = .84$	$R^2 = .82$	$R^2 = .81$

^at-values given in parentheses.

^bMeasured in hundreds of square-feet.

^cMeasured in hundreds of dollars.

Table 3.6. Implicit prices obtained under alternative functional forms

Variable name	Expected sign	Linear model	Semi-log model	Log-linear model	Box-Cox ^a model
F ^{*b}	(-)	\$ -11.38	\$ -21.03*	\$ -7.19*	\$ -11.63*
BED	(+)	3,524	5,222*	5,813*	6,363*
BATH	(+)	17,572*	8,765*	9,981*	8,733*
FAMRM	(+)	2,146	3,676	3,951	3,592
LOT ^c	(+)	147.50*	45.20*	26.82*	53.94*
DISH	(+)	4,517	8,276*	6,917*	8,088*
CA	(+)	-1,262	3,574	4,618*	3,641
GAR	(+)	4,425	11,059*	10,811*	11,243*
FP	(+)	-915	1,729	1,095	1,324
AGE	(-)	-125	-160*	-118*	-157*
NBHD ^d	(+)	31.69	40.24	21.14	28.31
SQFT	(+)	26.96*	16.38*	20.24*	17.29*
BASE	(+)	4,051	8,976*	10,637*	11,182*
LOC	(-)	-331	-240	-116.44	191

^aBox-Cox model was estimated with $\theta = -0.10$; $\lambda = 0.29$.

^bImplicit price listed is: $\frac{\partial P(H)}{\partial \$1} = \frac{\partial P(H)}{\partial F^*} \div \text{square feet of heated floor area.}$

^cMeasured in hundreds of square-feet.

^dMeasured in hundreds of dollars.

*Indicates implicit price is significant at a 95 percent confidence level.

prices of housing characteristics which are obtained under the four model specifications.

Table 3.6 reveals that the implicit price obtained for a given housing characteristic is sensitive to changes in the form of the hedonic price function. For example, the implicit price of an additional 100 square-feet of lot area (LOT) is over six times greater when a linear model is used to estimate the hedonic price function than when a log-linear model is used. Not only the magnitude of the implicit prices change under different model specifications, but in three cases the sign of the implicit price changes as well. The implicit price of an air-conditioner (CA), a fireplace (FP), and the distance of the home from the central business (LOC) have different signs under different model specifications. Halvorsen and Pollakowski (1981) stated that functional form misspecification is a potentially serious source of bias in hedonic price studies. The large degree of variation in implicit prices obtained under alternative functional forms in this study supports Halvorsen and Pollakowski's argument.

The value of the log likelihood function for the linear, semi-log, log-linear, and Box-Cox models is -2,645, -2,515, -2,508, and -2,502, respectively. A likelihood ratio test was performed to test the null hypothesis that the linear, log-linear, and semi-log models are significantly different from the Box-Cox model. The tests are based on the large sample theory that:

$$-2\log\psi = -2(\log L_{\Omega} - \log L_{\max})$$

where: L_{Ω} = the value of the likelihood function for the restricted
(linear, log-linear, or semi-log) model, and

L_{\max} = the maximum value of the likelihood function (obtained when
 $\theta = -0.10$ and $\lambda = 0.29$),

follows a chi-square distribution with k degrees of freedom, where k is the number of restricted parameters (Pindyke and Rubinfeld, 1981). In this case, since θ and λ are restricted for each model, $k = 2$. The null hypothesis that the restricted model is different from the unrestricted model cannot be rejected if:

$$-2(\log L_{\Omega} - \log L_{\max}) \leq \chi_{\alpha}^2$$

where: $\alpha = P(\chi^2 \leq \chi_{\alpha}^2)$. Using the test statistic $\chi_{.95}^2 = 5.99$, the null hypothesis that the linear, log-linear and semi-log models are different from the Box-Cox model at a 95 percent confidence level cannot be rejected.

Alternative specifications of the hedonic price function were examined in this section. First, the effect of changes in the included variables on the prices of housing characteristics was investigated. A full hedonic price function was estimated, containing the complete set of variables which were available for this study. A restricted model was estimated in which four of the structural variables which were not significantly different from zero¹ in the full model were excluded. It

¹At a 95 percent confidence level.

was found that the implicit prices of the housing characteristics in the restricted model were not substantially different than those in the full model. Next, the effect of changes in the functional form of the hedonic price function on the implicit prices of housing characteristics was examined. The linear, semi-log, and log-linear functional forms frequently used in hedonic studies were estimated, as well as the Box-Cox model. It was found that the implicit prices obtained using the linear, semi-log, and log-linear functional forms were substantially different than those obtained using the more flexible Box-Cox model. A likelihood ratio test revealed that it cannot be rejected that the linear, semi-log, and log-linear models are significantly different from the Box-Cox model at a 95 percent confidence level.

The following section will discuss the implicit prices obtained using the restricted Box-Cox model. The implicit prices of housing characteristics obtained in this model will be compared with those obtained in previous studies.

Model Results

Based on the exploration of alternative functional forms in the previous section, the Box-Cox model was chosen as the functional form which best fit the Des Moines housing sample used in this study. In this section, the implicit prices obtained using the Box-Cox model and the restricted list of explanatory variables will be discussed. The regression results of this model are shown in Table 3.5 and the implicit

prices which are calculated from this model are listed on both Table 3.4 and Table 3.6.

In examining the implicit prices obtained using the Box-Cox model, it should be noted that each of these implicit prices are obtained by evaluating the hedonic price function at the mean value of the independent variables. Because the function is non-linear, implicit prices evaluated at nonmean levels will differ.

Nine of the 14 independent variables in the Box-Cox model are significant at a 95 percent confidence level. Only one of the variables does not have the hypothesized sign. As shown in Table 3.5, the location variable, LOC, has a positive coefficient. This indicates that the value of a home increases with distance from the central business district, *ceteris paribus*. A positive relationship between price and distance from the central business district is contradictory to most economic theories of residential location. One possible explanation for this result is that residential preference in Des Moines is based on accessibility to employment centers other than the central business district. Another possible explanation is that the LOC variable is correlated with other excluded neighborhood variables, causing the LOC coefficient to be biased. In explaining the sign of the LOC coefficient, it should be noted, however, that the LOC variable is not significant in the hedonic model.

As explained in the theoretical discussion of the hedonic model, the implicit price of a housing characteristic is the increase in the expected selling price of the house due to a one unit increase in the

characteristic, *ceteris paribus*. Comparison of the implicit prices obtained in this study with the implicit prices obtained in other studies is difficult due to the variation in explanatory variables and functional forms used. Many of the previous studies have used census tract level data in estimating the implicit price function (Harrison and Rubinfeld (1977), Nelson (1978), and Godwin (1977)). These studies typically have numerous neighborhood characteristics and few structural, or house specific, characteristics. Studies based on observations of individual homes, rather than mean or median census tract values, have more structural characteristics included. Three such studies are Linneman (1980), Kain and Quigley (1970), and Johnson (1981). These three studies use different functional forms and exhibit a great deal of variation in the structural characteristics that are included in the implicit price function; however, a comparison of the results obtained in this paper will be made with these studies, whenever possible.

Using the Box-Cox model, the implicit prices obtained for the age of the home, the presence of central air conditioning, and the presence of a fireplace, have the same signs and are within the range of the values found in the Linneman, Johnson, and Kain and Quigley studies. The implicit prices of an additional bathroom, basement, garage, and lot size, however, vary substantially from some previous studies. Possible explanations for these variations will be considered.

For the Des Moines housing market, the implicit price of an additional bathroom is estimated to be \$8,733. In Linneman's combined Chicago and Los Angeles model, the implicit price of an additional

bathroom is \$9,457. In the Kain and Quigley model of St. Louis, the coefficient on the number of bathrooms variable is .036, i.e., an additional bathroom increases the value of the home by 3.6 percent. Evaluated at the mean price of homes in the Des Moines housing sample, this would indicate that an additional bathroom would increase the value of the home by \$2,391.

The implicit price of an additional bathroom obtained from the hedonic model of the Des Moines housing market is similar to the results of the Linneman model, but substantially higher than the implicit price of an additional bathroom found in the Kain and Quigley model. One possible explanation for the gap in these implicit price estimates is that the bathroom variable in both the Des Moines model and the Linneman model may be correlated with left out quality variables. It may be possible that homes having additional bathrooms also are of higher quality. In the Des Moines model and the Lineman model, no quality measures were included as independent variables. If quality is correlated with additional bathrooms, the coefficient on the number of bathrooms in these two studies could reflect the premium paid for additional quality as well as for an additional bathroom. In the Kain and Quigley model, five measures of quality are included in the hedonic price function; therefore, it is less likely that the implicit price of an additional bathroom in their study would reflect the affect of excluded quality variables.

The implicit price of the basement variable in the Des Moines hedonic price function indicates that a premium of \$11,182 is paid for

homes having a basement. Of the other three studies, only the Linneman model included a basement variable. The implicit price obtained by Linneman is \$1,760, only one-sixth of the implicit price obtained in the Des Moines hedonic price model. It is possible that the supply of and demand for basements is different in the Chicago and Los Angeles area than in the Des Moines area. Differences in the underlying supply and demand functions could cause a substantial difference in the premium paid for a basement in the three cities. The severity of the weather in the Des Moines area and the frequency of tornadoes may cause a high demand for basements. The fact that 92 percent of the homes in the Des Moines sample have basements indicates a high demand for them. The difference in the implicit price of a basement obtained in the two studies may also be due to the different independent variables included in the hedonic model or the different functional forms used.

As indicated in Table 3.4, the implicit price of a garage was estimated as \$11,243. This is similar to the \$8,234 implicit price of a garage obtained by Linneman in his Chicago-Los Angeles model, yet substantially greater than the values obtained by Johnson. Johnson estimated the implicit prices of one and two car garages in Knoxville to be \$1,580 and \$5,505, respectively. It is possible that the snow and wind of the Iowa winters cause the value of a garage to be higher in Iowa than it is in Tennessee. Again, differences in included independent variables and functional form may account for a portion of the difference in the implicit prices obtained.

The implicit price of an additional square-foot of lot size obtained in the Des Moines model is substantially more than in the Knoxville model (Johnson) and substantially less than in the St. Louis model (Kain and Quigley). The opportunity cost of the land may explain this variation. Des Moines and Knoxville are of equivalent size, however, Iowa land is much more fertile than Tennessee land. Since urban land must compete with agricultural uses, the opportunity cost of urban land in Iowa would be higher than in Tennessee. St. Louis is a much larger metropolitan area than Des Moines. The larger size of St. Louis may cause the demand for urban land there to be greater than in Des Moines. The implicit price of an additional square-foot of lot area, therefore, would be expected to be higher in St. Louis than in Des Moines.

None of the other studies estimated the implicit price of a dishwasher. The implicit price of a dishwasher obtained in this study is \$8,088. Intuitively, this seems to be an unreasonably high value. A likely explanation is that the implicit price obtained reflects not only the value of a dishwasher, but also the value of other excluded variables. Homes having dishwashers may be more likely to have other kitchen appliances and quality kitchen cabinets or countertops. A positive correlation between the dishwasher variable and these excluded variables would cause the coefficient on DISH to be biased upward, reflecting the entire value of a modern kitchen, rather than just the value of a dishwasher.

Finally, the variable of key interest in the hedonic model is the implicit price of fuel expenditures. The implicit price of \hat{F}^* , predicted

fuel bills per square-foot, is -\$25,466.58, i.e., a \$1 decrease in predicted fuel bills per square-foot increases the value of the house by \$25,466. Dividing this value by the mean value of square-feet of heated floor area reveals that a \$1 decrease in fuel expenditures per year (due to an increase in heating efficiency rather than an increase in the internal temperature of the home) results in an \$11.63 increase in the selling price of the home. A word of caution must be added to the interpretation of the implicit price of \hat{F}^* . As discussed in the data section of this chapter, no true measure of the structural efficiency of the sample homes was available. \hat{F}^* was designed to reflect the component of each household's actual fuel bills which was due to the structural efficiency of the home. The implicit price of \hat{F}^* in this study is only successful in reflecting the implicit price of an increase in energy efficiency to the extent that \hat{F}^* is an accurate measure of efficiency.

Some investments which increase the winter heating efficiency of a home increase the summer cooling efficiency of the home as well (e.g., ceiling insulation). A relevant question is whether the implicit price of a one dollar decrease in annual winter fuel expenditures is equal to the present value of the dollar's worth of winter fuel savings alone, or whether the implicit price includes the present value of summer cooling savings (caused by the increase in winter heating efficiency) as well.

To explore this issue, the hedonic model was estimated with an interaction term between \hat{F}^* and CA (a variable indicating the presence of central air-conditioning) was included as an independent variable.¹ If

¹ $\hat{F}^* * CA$ was included in the regression equations.

home purchasers include summer cooling effects in the premium that they are willing to pay for an increase in winter heating efficiency, then it was expected that the coefficient on the interaction term would be negative and significant, i.e., home purchasers using summer cooling will be willing to pay a higher premium for energy efficient homes than buyers purchasing homes which do not have summer cooling. The coefficient on the interaction term was negative, however, it was not significant at a 95 percent confidence level.¹ Based on these results, it could not be concluded that the implicit price of \hat{F}^* includes the present value of summer cooling benefits as well as the present value of one dollar's worth of winter fuel savings.

In interpreting the implicit price of \hat{F}^* it must be recalled that the hedonic price function is nonlinear. The implicit price was obtained by assuming that the value of predicted fuel bills per square-foot, \hat{F}^* , and all other characteristic levels were at their mean values. As previously discussed, an advantage of the Box-Cox model is that it allows the implicit price of a housing characteristic to rely on the level of the characteristic itself, as well as the level of other housing characteristics. Further information about the relationship between housing prices and energy efficiency levels may be obtained by examining how the implicit price of energy efficiency is altered as the level of \hat{F}^* and other characteristics deviate from their mean values.

The first derivative of the hedonic price function with respect to \hat{F}^* indicates a negative relationship between fuel bills per square foot

¹The t-value of the interaction coefficient was equal to -1.17.

and housing prices, i.e., $\frac{\partial P(H)}{\partial \hat{F}^*} < 0$. The second derivative of the hedonic function with respect to \hat{F}^* is positive, i.e., $\frac{\partial^2 P(H)}{\partial \hat{F}^{*2}} > 0$. A negative first derivative and a positive second derivative indicate that the relationship between housing prices and fuel bills per square-foot is decreasing at an increasing rate. This relationship is pictured in Figure 3.2.

The increase in $P(H)$ due to a decrease in fuel bills per square-foot is greater at high levels of fuel bills per square-foot than at low levels of fuel bills per square-foot. At the average efficiency level of sample homes, a one dollar decrease in annual fuel expenditures is expected to increase the resale value of the home by \$11.63. In a very

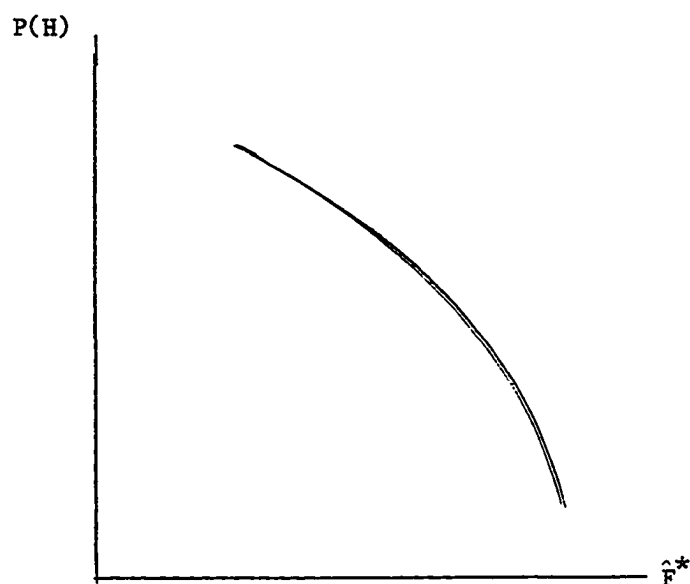


Figure 3.2. Relationship between energy efficiency and housing prices

inefficient home, a one dollar decrease in annual fuel expenditures will increase the home's resale value by more than \$11.63 and in a very efficient home, a one dollar decrease in annual fuel expenditures will increase the resale value of the home by less than \$11.63. This result implies that increases in efficiency are valued more in inefficient homes than in efficient homes. There may be a level of fuel expenditures per square-foot which is considered "reasonable." Buyers will offer less for a home which has a level of fuel expenditures greater than this "reasonable" level, *ceteris paribus*; however, they will not be willing to pay a high premium in order to obtain a home having a level of fuel expenditures much less than this "reasonable" level.

By examining $\frac{\partial^2 P(H)}{\partial \hat{F}^* \partial \text{Age}}$, the relationship between the implicit price of energy efficiency and the house's age may be obtained. $\frac{\partial^2 P(H)}{\partial \hat{F}^* \partial \text{Age}} < 0$, indicating that the increase in $P(H)$ due to a decrease in \hat{F}^* (i.e., an increase in efficiency) is less in older homes than in newer homes. There is no a priori reason to assume that energy efficiency is valued more highly in newer homes than in older homes. It is possible, however, that on the average, the age of fuel saving investments in older homes is greater than the age of fuel saving investments in newer homes. For example, in a house that is two years old, no fuel saving investment (i.e., insulation, or high efficiency furnace) could be more than two years old. In a house that is 30 years old the fuel saving investments may be up to 30 years old. If each fuel saving investment has a finite expected life, say 50 years, then the remaining life of the fuel saving

investments in the two year old home is much longer than the remaining life of the fuel saving investments in the 30 year old home. Due to the additional remaining life of the fuel saving investments in the two year old home, the premium that a buyer would be willing to pay for this home would be greater than the premium that a buyer would be willing to pay for the 30 year old home.

Only one of the three comparison studies included fuel expenditures in the hedonic price function. The Johnson study of the Knoxville housing market has a predicted fuel bill variable included as a right hand side variable. Her study results indicate that the implicit price of increases in fuel expenditures is $-\$20.73$, i.e., a $\$1$ increase in annual fuel expenditures will decrease the selling price of the home by $\$20.73$, *ceteris paribus*.

The difference between the value of the implicit price obtained in this study and in Johnson's study could be due to the fact that Johnson constrained the hedonic price function to be linear. As indicated in the model estimation section of this chapter, the implicit price of a given housing characteristic may vary significantly with changes in functional form. The Box-Cox model used to estimate the hedonic price function in this study allows the implicit price of energy efficiency to depend on the level of efficiency itself. It was demonstrated that the implicit price of an increase in efficiency increases as \hat{F}^* increases. If the decrease in $P(H)$ due to an increase in \hat{F}^* is much greater at high levels of \hat{F}^* than at low levels of \hat{F}^* , then the slope obtained when this

function is forced to be linear could be greater than the value of $\frac{\partial P(H)}{\partial \hat{F}^*}$ obtained at the mean value of \hat{F}^* .

By comparing Johnson's estimate with the implicit price of fuel expenditures obtained when the Des Moines model is constrained to be linear, this hypothesis may be explored. Table 3.5 indicates the implicit prices of housing characteristics obtained when the hedonic price function of Des Moines is constrained to be linear. The implicit price of a \$1 decrease in fuel expenditures in this case is \$11.38.¹ It cannot be concluded, therefore, that the linearity constraint imposed by Johnson is solely responsible for the differences in the implicit price of fuel savings in the two studies.

Differences in the underlying supply and demand functions for energy efficient homes in Knoxville and Des Moines may also account for a difference in the implicit price of fuel expenditures in these two cities. Over 90 percent of the homes in the Des Moines study area are heated by natural gas (Clark Bruebaker, Iowa Power, Des Moines, personal communication, 1984). In the Knoxville area, approximately 70 percent of the homes are heated by electricity (Johnson, 1981, p. 46). If the expected rate of electricity price increase in Knoxville is higher than the expected rate of natural gas price increase in Des Moines, then the present discounted value of \$1 worth of fuel savings may be greater in Knoxville than in Des Moines. Alternatively, if the supply of energy

¹In the linear model, the coefficient of \hat{F}^* is not significant at a 95 percent confidence level.

efficient homes in Knoxville is limited, then the premium paid for energy efficiency may be greater than in Des Moines.

The estimate of $\frac{\partial P(H)}{\partial \hat{F}^*}$ which is obtained in the hedonic model of the Des Moines housing market is of great importance to individuals planning to build a home which they will only own for a limited period of years. These individuals may not live in the home long enough to have fuel saving investments, such as insulation, tight construction, and high energy efficiency furnaces, pay off during the time they own the home. In deciding whether or not to include them in the home, therefore, they need to be able to estimate the impact that these investments will have on the selling price of the house. The implicit price obtained from the hedonic price model may be used to estimate the resale value of each fuel saving investment. These resale values may then be used in determining the efficient mix of fuel saving investments for individuals who plan to own their home for a number of years less than the life of the fuel saving investments.

In this chapter, the relevant issues in the construction of hedonic models were discussed. The data and model estimation procedure used in this study were examined. In light of previous studies, a Box-Cox procedure was chosen to estimate the hedonic price function for the Des Moines housing sample. Two Box-Cox models, having alternative specifications of independent variables, were estimated. A full model contained all the available information on the sample housing characteristics, and a restricted model was estimated using a reduced number of these housing characteristics. In general, the difference

between the implicit prices in the two models is small (less than ten percent for the majority of housing characteristics).

For comparison purposes, a linear, semi-log, and log-linear model of the hedonic price function were estimated. The implicit prices obtained using these models varied substantially from the implicit prices obtained using the Box-Cox model. A likelihood ratio test revealed that the Box-Cox model fit the data significantly better than any of the alternative models.

The key variable of interest in the hedonic model is the implicit price of \hat{F}^* . \hat{F}^* is a proxy variable representing a true measure of heating efficiency and is obtained by an instrumental variable procedure. The results of the hedonic model reveal that a \$1 decrease in predicted fuel expenditures (due to an increase in efficiency) increases the expected selling price of the home by \$11.63. This implicit price may be used to estimate the resale value of fuel saving investments. In the following chapter, the method used to calculate the resale values is examined. Information on the resale value of fuel saving investments is necessary for a household which does not plan to own its home over the entire life of the investment. A linear programming model is used to obtain the efficient mix of fuel saving investments for a baseline home under alternative scenarios. The cost minimizing investment mix obtained when the home is to be owned over the life of all the fuel saving investments (a 50 year period) will be compared with the cost minimizing investment mixes obtained when the home is to be sold at various times during the life of the fuel saving investments.

CHAPTER 4. LINEAR PROGRAMMING MODEL

There are many fuel saving investments which may be incorporated into newly constructed homes. Insulation, passive solar applications, high energy efficiency furnaces and reduced air changes per hour may all be incorporated at the time of construction. Each of these investments increase the fixed cost of building the home and reduce the fuel expenditures necessary to maintain the home at the desired temperature level. This chapter discusses a linear programming approach to determining which mix of these fuel saving investments is "optimal." As discussed in the theory chapter, the consumer chooses the level of temperature which maximizes his utility, and then attempts to achieve this temperature level at a minimum cost. Once cost minimization is defined as the criteria to be used for selecting among alternative investments, a linear programming model is a useful tool for determining the "optimal" fuel saving investment mix.

In the first section of this chapter, the strengths and weaknesses of linear programming are discussed. The objective of the linear programming model in this study is examined in the second section. The third section discusses the linear programming model and its constraints, activities, and underlying assumptions. The method by which the cost coefficients and energy providing coefficients for the model are obtained are described in the fourth section. The optimal activity mix for each alternative time period is examined in the fifth section, along with a

discussion of the relevant penalty costs and opportunity costs. The stability of the solutions obtained are examined in the final section.

A Discussion of the Strengths and Weaknesses of Linear Programming Models

Linear programming was first applied to planning problems during World War II and in the post war period. Since then it has been used in a number of different contexts. Linear programming has been useful in determining least cost feed rations for livestock, optimal locations for production and warehouse facilities, production schedules to achieve the greatest output from a given production facility, and numerous other issues. In this paper, a cost minimizing linear programming model is used to determine the most efficient method of maintaining the desired temperature level in a home. The linear programming model is able to consider fuel saving investments simultaneously in order to determine which of these provide the desired temperature level at a minimum cost. It is also able to incorporate the time constraints and personal preferences of the owner as well as the existence of tax credits and housing codes. The model can be used to determine how the cost minimizing investment mix would be altered under different rates of fuel price increase, alternative tax codes, and different internal temperature levels.

In using a linear programming model, its limitations must be examined as well as its strengths. As the name "linear" programming indicates, the model assumes linear relationships between inputs and

outputs. Extra steps must be taken to deal with nonconstant input-output relationships, such as diminishing marginal returns or decreasing costs. Several of the activities included in the home heating linear programming model considered in this paper exhibit decreasing marginal returns. For example, as insulation is added to the ceiling or walls of a house, the savings in natural gas consumption are greater for initial increases in R-value than for later ones. The limitation imposed by the assumption of linear input-output relationships is circumvented in the home heating model by subdividing activities with nonlinear input-output relationships into several separate activities. By this method, nonlinear production relationships may be approximated by the use of several activities. Because the linear programming model is not able to estimate input-output relationships, the necessary production relationships were obtained using a computer simulation model.

A second limitation of a linear programming model is that it cannot forecast price expectations. All current and future prices must be inputted into the model. The accuracy of the model is limited by the accuracy of the prices included in it (Beneke and Winterboer, 1973). In the home heating model, the price of natural gas is the key unknown which must be included in the model. If the future prices used in the model underestimate the actual price increase, then the model solution will exhibit an underutilization of fuel conserving investments. Likewise, if the future prices used overestimate the actual price increase, then the solution will show an overinvestment in fuel conserving activities. Finally, as previously discussed, the linear programming solution

obtained is "optimal" only in the sense that it minimizes the cost of maintaining the house at a given temperature. In the initial analysis, it is assumed that the house is to be maintained at 68° F throughout the heating season. This temperature, however, is exogenous to the model and the programming solution may change if the desired temperature level is altered. If the temperature level which is exogenously assigned to the linear programming model is not the utility maximizing temperature level chosen by the homeowner then the programming solution may not be optimal. To examine the magnitude of this problem, sensitivity analysis is used to determine how sensitive the programming solution is to changes in the desired temperature level.

Objective of the Linear Programming Model

As discussed in the introduction, two key questions addressed in this study are: What mix of fuel saving investments is most efficient for a homeowner to incorporate into a newly constructed home? How does the efficient investment mix change as the period of time the individual plans to own the home is altered? In order to address these two questions, it is necessary to define a method of obtaining an efficient investment mix under alternative time horizons.

The discussion in the theory chapter revealed that each household may be viewed as a small firm producing its desired internal temperature level at a minimum cost. It was demonstrated that a cost minimizing linear programming model may be used to obtain the efficient fuel saving investment mix for a given household. In this study, a single-story

house located in Des Moines, Iowa is defined as a baseline home. It is assumed that the household desires to maintain the internal temperature of the baseline home at 68° F. The linear programming model generates the efficient investment mix for the baseline home under alternative time horizons.

First, the linear programming model is used to obtain the efficient mix for the home under the assumption that the house will be owned for 50 years. It is assumed that insulation, passive solar applications, and reduced air changes per hour have an expected life of 50 years. The expected life of a furnace is assumed to be 20 years. When cost is minimized over a 50 year period, therefore, the entire flow of benefits from each investment is included and the long run total cost of heating the home is minimized.¹

A new homeowner may not plan to live in the home for a 50 year period, however, and may not wish to minimize the long run total cost of heating the home. He will rationally plan to minimize the portion of the home's heating cost which he will incur. For each investment, he will want to consider its initial cost, the fuel savings it brings about during the period he owns the home, and its resale value.² In Des Moines, Iowa (where the baseline home is located), the average home is owned for a six year period. To obtain the cost minimizing investment mix for a homeowner who plans to own his home for a six year period, the

¹The investment criterion in this case is: $C \leq \int_0^T MB(t)e^{-rt} dt$.

²The investment criterion in this case is:
 $C \leq \int_0^n MB(t)e^{-rt} dt + (P_2 - P_1)e^{-rn}$.

linear programming model is run using a six year time horizon and incorporating the resale value of each fuel saving investment.

In order to estimate the resale value of each fuel saving investment, it is necessary to determine how the housing market evaluates energy efficiency. The hedonic model (as described in the previous chapter) provides an estimate of the implicit price of energy efficiency for homes in the Des Moines area, i.e., the expected increase in the selling price of a home due to an increase in efficiency, *ceteris paribus*. The implicit price of energy efficiency obtained from the hedonic price model is used to estimate the resale value of each of the fuel saving investments considered in the linear programming model.

Finally, the linear programming model is run using a 20 year time horizon. The solution to this model indicates the efficient investment mix for an individual who plans to sell his home after 20 years. The 20 year period was chosen because it was felt that the resale values calculated in this study may be most accurate when it is assumed that the fuel saving investments are 20 years old. The implicit price obtained from the hedonic price model is used to estimate the resale value of each fuel saving investment. In calculating this implicit price, it is assumed that each housing characteristic is at its average level in the Des Moines housing sample. This implicit price, therefore, is valid for fuel saving investments which are the average age of fuel saving investments in the Des Moines housing sample. Unfortunately, the average age of fuel saving investments is not known. It is known, however, that the average age of houses in the sample is 30 years old. If all fuel

saving investments had been installed at the point of construction, then the average age of fuel saving investments would be 30 years old as well. A more realistic possibility is that not all fuel saving investments were installed at the point of construction and that the average age of fuel saving investments is less than the average age of homes. In light of the fact that the average of homes in the Des Moines sample is 30 years old, it is hypothesized that the average age of fuel saving investments in the housing sample is twenty years old. Based on this hypothesis, the resale value of each fuel saving investment is most accurate when it is assumed that the investment is 20 years old when it is sold.¹

Once the efficient fuel saving investment mixes for the 50, 20, and 6 year time horizons are obtained, these solutions may be compared to determine how the efficient investment mix changes when the length of time the home is to be owned is altered. The following section discusses the specifics of the linear programming model used in this paper, the fuel saving activities which are considered in the model, and the constraints which are imposed on the model's solution.

Objective Function, Activities, and Constraints, of the Linear Programming Model

The objective of the linear programming model is to minimize the cost of maintaining a baseline house at 68 degrees Fahrenheit over the relevant planning horizon.

$$\text{Min } \sum_i N_i^j x_i, \quad (4.1)$$

¹A further discussion of this point may be found in Appendix C.

- S.T. (1) $x_i \leq Z^*$,
 (2) design preferences,
 (3) constructional feasibility and tax codes.

where: N_i^j = the net cost (initial cost - present value of summer savings - present value of resale value) of each activity i ,
 assuming that the home is sold in year j ,
 x_i = activity (investment) i which is used to produce heat,
 q_i = the quantity of heat obtained from x_i , and
 Z^* = the BTUs of energy necessary to maintain the house at 68° F over the relevant time horizon.

As defined, Z^* , is the BTUs of energy necessary to maintain the house at 68° F over the relevant time horizon. In order to calculate Z^* , both the characteristics of the house and the time horizon must be defined. In this study, a single-story house located in Des Moines, Iowa was defined as a baseline home. The characteristics of this baseline home may be found in Table 4.1. Three different time horizons; 50, 20, and 6 were used in obtaining model solutions. Once the house, temperature, and time horizon were specified, the BTU requirement, Z^* , was determined by using heat loss equations (Hodges, 1980). The linear programming model was then used to determine the mix of activities which provide the BTU requirement at a minimum cost. Four basic technologies for providing the home's energy needs were considered:

- (1) Two passive solar activities were considered. In Passive Solar I, south glass may be increased up to ten percent of the total floor area

Table 4.1. Characteristics of the baseline home^a

-
- a 40 foot x 26 foot rectangular, single story house with:
 - 1,040 square feet of above ground floor area with 8 foot high walls
 - 1,040 square feet of basement floor area with 8 foot high walls
 - moderately insulated with:
 - R-30 level ceiling insulation
 - R-19 level wall insulation
 - no basement insulation
 - double paned windows
 - one air change per hour (ACH) in the first floor area
 - 1/2 air change per hour in the basement area
 - volume of house = 16,640. The volume was adjusted to 12,500 in the computer simulation model to reflect the 1/2 ACH in the basement area
 - thermostat setting assumed to be 68° F
 - 100,000 BTUs of internal heat gain assumed
 - .975 air density ratio
 - geographic setting:
 - located in Des Moines, Iowa
 - 6,550 heating degree days per heating season
 - home has a due south facing wall (Azimuth - 0°)
 - south wall is perpendicular to ground (tilt factor = 90°)
 - absorbance of south wall = .75
-

^aRecommended by Pat Huelman, Energy Extension Specialist, Iowa State University, Ames, Iowa.

of the home. In Passive Solar II, south glass may be increased between the range of 10 and 20 percent of the total floor area, however, the additional south glass must be accompanied by additional mass to store the incoming radiation.

(2) Increasing insulation in the ceiling, above-grade walls, basement walls, window headers and floor joist were all considered as means of providing the BTU requirement. Night time insulation on the additional south glass was also considered.

(3) Reducing air changes per hour (ACH) in the home is possible by increasing air infiltration control, using increased care in construction, and utilizing heat recovery units. Four different options were included in the model to reduce the air changes per hour to .5, .375, .3, or .25.

(4) Five different furnaces were included in the model, ranging from 95 percent annual fuel utilization efficiency (AFUE) to 65 percent AFUE.

Other activities in the model included obtaining applicable tax credits and adding nonsouth glass to the home. A more detailed description of each activity may be found in Table 4.2.

Several constraints were incorporated into the linear programming model. As indicated in equation 4.1, the model was constrained to provide the minimum BTU requirement necessary to maintain the baseline home (see Table 4.1) at 68° Fahrenheit over the relevant planning horizon. Further, the model was constrained to include no more than the maximum amount of south glass or insulation considered constructionally

Table 4.2. Activities in the linear programming model

Listed below are the energy consumption reducing and energy providing activities which were examined in the model as means of providing the BTU requirement:

- Passive Solar I: Increase south glass up to 10% of the total floor area
 - Passive Solar II: Increase south glass from 10% of the floor area up to a possible 20% of the total floor area. Glass area increases in this range include adding additional amounts of mass to the house structure in order to store the incoming radiation
 - Add R-4 level insulation to the chosen amount of south glass
 - Add R-6 level insulation to the chosen amount of south glass
 - Increase ceiling insulation by an additional R-value up to a maximum R-60 level
 - Increase above grade wall insulation up to a maximum R-65 level
 - Basement Insulation I: Add R-5 insulation to the two feet of above grade basement wall
 - Basement Insulation II: Add R-5 level insulation to the two feet of above grade basement wall and the top four feet of below grade basement wall
 - Basement Insulation III: Add R-5 level insulation to the two feet above grade basement wall and the entire six feet of below grade basement wall
 - Basement Insulation IV: Add R-10 level of insulation to the two feet of above grade basement wall and the top four feet of below grade basement wall. Add R-5 level insulation to the lowest two feet of basement wall
 - Increase window header insulation from an R-9 level to a R-22 level
 - Increase floor joist insulation from an R-8 level to a R-27 level
 - Add one square foot of nonsouth glass
 - Decrease the air changes per hour (ACH) to .5 ACH by sealing joints, adding foam to cracks, and using more care in construction
 - Decrease ACH to .375 by increasing infiltration control and adding a window heat recovery unit (HRU)
 - Decrease ACH to .30 by tight infiltration control, care in construction, and use of two HRUs
 - Decrease ACH to .25 by tight infiltration control, care in construction, and use of one large HRU and a distribution system with 70% efficiency
 - Purchase a 95% annual fuel utilization efficiency (AFUE) furnace
 - Purchase a 86% AFUE furnace
 - Purchase a 84% AFUE furnace
 - Purchase a 78% AFUE furnace
 - Purchase a 65% AFUE furnace
 - Buy natural gas
-

feasible. A minimum amount of nonsouth glass was forced into solution as an aesthetic preference for windows on the nonsouth walls. Finally, two constraints were used to limit the amount of tax credits which may be obtained on fuel saving investments. A 15 percent Energy Saver Tax Credit is available for investments in insulation and a Solar Tax Credit is available for single purpose solar investments (Catherine Sibold, Energy Policy Center, Des Moines, Iowa, personal communication, 1981). The Energy Saver Tax Credit utilized in the model was constrained to \$300, the maximum amount allowable under U.S. tax laws, and the Solar Tax Credit was limited to \$4,000, its maximum allowable amount (Edward Roach, Internal Revenue Service, Tax Credit Division, personal communication, 1981).

The Data

Unless the physical and economic environment are defined, an "optimal" fuel consumption reducing investment mix does not exist. What is "optimal" varies with the region of the country, the severity of the winter, the amount which the price of natural gas is expected to rise, and the discount rate used to evaluate future costs. To obtain an "optimal" investment mix, one must make assumptions about each of these factors. In this study, weather data corresponding to an average winter in Des Moines, Iowa (i.e., a heating season with 6,550 heating degree days) was used. It was assumed that the price of natural gas will increase at a real rate of five percent annually and that future costs

are discounted by a five percent real discount rate.¹ The amount which the price of natural gas will actually increase is, of course, unknown. The magnitude of this increase, however, will affect the economic viability of the fuel saving activities in the model and may affect the programming solutions. To address this issue, the price of natural gas was parameterized and optimal solutions were obtained for various annual real rates of fuel price increase ranging from 0 to 9 percent. This process is discussed in the sensitivity analysis section of this chapter.

Note that it was assumed that natural gas, rather than electricity or fuel oil, was used to meet the home's heating needs not met by insulation, passive solar or reduced air changes. Natural gas was chosen as the auxiliary heat source for two reasons: (1) over 90 percent of the homes in the study area use natural gas to meet their heating needs (Clark Brubaker, Iowa Power, Des Moines, personal communication, 1984); and (2) currently natural gas is a cheaper fuel source than either electricity or oil in the Des Moines area, and price projections indicate that it should remain cheaper out to the year 2020 (Office of Policy, Planning and Analysis, 1983).

¹According to the Office of Policy, Planning, and Analysis (1983), the real price of natural gas in the Des Moines area should rise by approximately four percent annually out to the year 2010. An alternative price forecast by Data Resources Incorporated (1980) predicted a nine percent annual real rate of price increase. In light of these two price projections, a five percent real rate of price increase was used in the initial analysis. A five percent real discount rate was chosen to reflect the long-run real cost of borrowing.

Obtaining the Model Coefficients

Once the BTU requirement necessary to maintain the home at 68° F, the activities, and the constraints of the model were defined, it was necessary to determine the cost and the energy provided by each activity considered. The cost information was incorporated into the objective function and the energy supplied by each activity provided the coefficient, i , in the constraint specifying the minimum energy requirement necessary to maintain the house at the desired temperature level (constraint 1 in equation 4.1).

Information on the BTUs of energy provided by each of the activities was obtained by use of a computer correlation model designed by Michael Ried at Iowa State University in 1981 (based on Balcomb's Solar Design Handbook: Passive Solar Design Analysis). The correlation model calculates the annual auxiliary heat requirement of a home as a function of the home's dimensions, volume, number of windows and doors, insulation levels, amount of south glass, type of passive system, geographic location and weather factors. By varying these characteristics individually, their impact on the auxiliary heat requirement of the house was obtained. Specifically, the correlation model was iterated with parametric changes in each relevant activity level, holding all other characteristics constant. This process yielded several different estimates of the auxiliary heat requirement corresponding to various levels of each characteristic (i.e., insulation level, ACH, amount of south glass). These estimates were then graphed and linear regressions were performed to obtain the slope of each relationship. Each slope

indicates the BTUs of energy provided by one unit of the given activity (i.e., the savings in auxiliary heat purchases made possible by one unit of the activity). Each slope was then used as a coefficient, β_i , in the minimum energy requirement constraint.

In cases in which the relationship between the activity level and auxiliary heat requirement was nonlinear, the relationships were approximated by linear segments and the activities were subdivided to reflect the different impact a per unit change has at various levels. For example, as ceiling insulation is increased the reduction in auxiliary heat requirement that a one unit change brings about decreases. To reflect this relationship the activity of adding ceiling insulation to the home was subdivided into two activities, adding insulation at levels less than R-32 and adding insulation at levels greater than R-32. These two activities have different coefficients to reflect the different energy savings a one unit increase in ceiling insulation has at high and low initial levels. For relationships which were nonlinear and had to be approximated by linear segments, numerous iterations were performed. Fewer observations were used in relationships which were clearly linear. Table A.1 (in Appendix A) indicates the number of observations used and slope coefficients obtained for each activity.

Several activities in the model were evaluated on a nonincremental basis. For example, the possibility of adding insulation to the basement was analyzed by considering four possible basement insulation options, rather than on a per unit basis. For activities evaluated in this way, the cost coefficient represents the cost of the entire activity and

the energy provision coefficient represents the energy provided by the entire option. A complete list of activities evaluated in this way, and their energy provision coefficients, may be found in Table A.2, Appendix A.

The initial cost associated with each activity reflects the value of the additional materials and labor required to include the activity into the housing structure. The initial cost estimate for each activity may be found on Table 4.3.

Estimating the cost of a passive solar system is difficult because the system is an integral part of the structure of the house. Only the additional construction costs of the solar system were included, i.e., the cost of the thermal storage floor minus the cost of a "regular" floor, plus the cost of a double glazing on the south wall minus the cost of building a conventional wall.

Note that in the objective function of the linear programming model (equation 4.1) the cost figure used for each investment is a "net cost" value. The net cost value of each investment differs from the initial investment cost in that it incorporates both the summer cooling benefits derived from the investment, and the resale value of the investment.

The linear programming model was designed to minimize total heating costs, however, some of these fuel saving investments provide summer cooling benefits as well. Disregarding the cooling benefits associated with each activity could result in an underinvestment in fuel saving devices. In order to incorporate the summer cooling benefits into the linear programming model, the initial cost coefficient for each activity was adjusted to reflect the present value of summer cooling benefits

which are obtained from it. The method used to calculate the summer cooling benefits of each investment may be found in Appendix B.

Similarly, an underinvestment in fuel conserving activities would result if the resale value of each of these activities was disregarded. It was assumed that the passive solar, reduced air changes, and increased insulation activities have an expected life of 50 years. If the house is to be sold in less than 50 years, therefore, the resale value of each of these investments must be considered when determining the optimal investment mix. It was assumed that the furnace has an expected life of 20 years. If the house is to be sold before 20 years, therefore, the resale value of the furnace must be considered as well. Failure to include the resale value of each investment in the analysis may result in an underinvestment in fuel conserving activities.

The resale value for each fuel saving investment was calculated using the implicit price of energy efficiency obtained from the hedonic model. A three step procedure was used to calculate the resale value of each investment:

Step 1: The decrease in natural gas consumption caused by each fuel saving investment was calculated:

For example, increasing the above grade wall insulation from an R-25 to an R-35 level reduces the auxiliary heat requirement of the baseline house by 1.12028 MBTU/year. Assuming that the auxiliary heat source is a 75 percent efficient gas furnace, the

increase in wall insulation reduces the amount of natural gas consumed by 13.595 CCFs per year.

Step 2: The dollar value of savings due to the decrease in natural gas consumption was calculated:

Assuming the price of natural gas is 0.60/CCF, the 13.595 CCF decrease in the amount of natural gas required annually results in a savings of \$8.16/year.

Step 3: Based on the dollar value of annual savings due to a fuel conserving investment and the implicit price from the hedonic model, the resale value of the fuel saving investment was calculated:

The implicit price of fuel expenditures reveals that a \$1/year decrease in fuel expenditures results in an \$11.63 increase in the selling price of the home. The \$8.16/year decrease in fuel expenditures brought about by the increase in above grade wall insulation, therefore, results in a \$94.87 (11.63×8.16) increase in the selling price of the house. If the house is sold after six years, the present value of the \$94.87 increase is \$70.80. If the house is sold after 20 years, the present value of the increased value of the house is \$35.75.

By using this method, the resale value of each fuel saving investment in the linear programming model was obtained. Table 4.3 indicates the resale value of each fuel saving investment calculated under the assumption that it is sold after six years, and that it is to

Table 4.3. Resale values of fuel saving investments

Investment	Initial investment Cost (C _i)	Resale values assuming house is sold after:	
		6 years	20 years
Passive Solar I (per sq ft) ^a	\$ 5.00	\$ 1.37	\$ 0.69
Passive solar II (per sq ft) ^a	10.00	0.74	0.37
R-4 night insulation ^b	429.00	243.70	123.09
R-6 night insulation ^b	220.00	220.00 ^c	163.07
Increase 1 R-value of ceiling insulation up to R-30 ^a	10.40	10.40 ^c	10.40 ^c
Increase 1 R-value of ceiling insulation up to R-60 ^a	10.40	6.61	3.34
Increase 1 R-value of above grade insulation R-16 to R-25 ^a	7.15	19.91	10.06
Increase above grade insulation to R-35 ^a	786.50	70.80	35.75
Increase above grade insulation to R-45 ^a	856.00	141.48	71.51
Increase above grade insulation to R-55 ^a	929.50	212.37	107.27
Increase above grade insulation to R-65 ^a	1,001.00	283.16	141.97
Basement Insulation I ^a	123.00	172.76	87.26
Basement Insulation II ^a	554.40	314.31	158.75
Basement Insulation III ^a	686.40	338.64	171.04
Basement Insulation IV ^a	819.40	480.81	242.85
Decrease air changes to .5 ^a	375.00	947.89	470.70
Decrease air changes to .375 ^a	625.00	1,485.00	750.23
Decrease air changes to .3 ^a	875.00	1,660.00	838.42
Decrease air changes to .25 ^a	1,125.00	1,776.00	897.01
Insulate window headers ^a	3.25	18.05	9.32
Insulate floor joist ^a	26.40	26.40 ^c	26.40
Install a 95% AFUE furnace ^d	2,200.00	383.73	0
Install a 86% AFUE furnace ^d	1,400.00	214.81	0
Install a 84% AFUE furnace ^d	1,300.00	175.76	0
Install a 78% AFUE furnace ^d	1,200.00	97.62	0
Install a 65% AFUE furnace ^d	1,000.00	0	0

^aInitial cost estimate obtained from Robert Buck, Buck Construction Company, Inc., Ames, Iowa.

^bInitial cost estimate obtained from Rhys Christenson, Christenson Building Services, Ames, Iowa.

^cIndicates initial installment cost used as an upper limit on the resale value.

^dInitial cost estimate obtained from Tom Greiner, Extension Agricultural Engineer, Cooperative Extension Service, Iowa State University, Ames, Iowa.

be sold after 20 years. It was assumed that the resale value of each investment is zero after 50 years.

The resale values obtained by the method outlined above have several limitations: (1) it was assumed that each resale value is based only on the winter fuel savings due to the investment and not the summer cooling benefits that may be obtained; (2) the implicit price used in calculating each resale value represents the implicit price of energy efficiency for a house at the average efficiency level of homes in the Des Moines housing sample. The actual implicit price and, therefore, the resale value, may differ for a home of greater or lesser efficiency; and (3) the implicit price of energy efficiency used in calculating the resale value of each fuel saving investment is based on the assumption that the fuel savings investments are the average age of fuel saving investments in the housing sample. The resale values of investment which are not the average age, therefore, may differ from the estimates obtained in this study. A detailed discussion of each of these limitations may be found in Appendix C.

Once the initial cost of each investment, the present value of the summer savings it brings about, and the present value of its resale value were determined, the "net cost" value could be calculated:

$$N_i^j = C_i - S_i^j - R_i^j \quad (4.2)$$

where: N_i^j = the net cost of investment i assuming that it is sold in year j ,

C_i = the initial construction cost of investment i

S_i^j = the present discounted value of j years worth of summer cooling savings due to investment i .

R_i^j = the present discounted value of the resale value of investment i , assuming that it is sold in year j .

Table 4.4 indicates the net cost of each investment under the assumption that the investment is sold after 6, 20, and 50 years alternatively. The net cost of each investment is the cost figure which was used in the objective function (equation 4.1) of the linear programming model.

Programming Solutions

The three alternative programming solutions obtained using the linear programming model are shown in Table 4.5. "Solution-50" represents the fuel saving investment mix which was obtained when the cost of heating the baseline home was minimized over a 50 year period. This investment mix minimizes the long run total cost of heating the home. In this solution, a 95 percent annual fuel utilization efficiency furnace (AFUE), and maximum insulation levels are installed in the home. The number of air changes per hour (ACH) are reduced to .25, the minimum number that was considered constructionally feasible. Neither of the passive solar activities (Passive Solar I and Passive Solar II) enter the model solution.

In obtaining Solution-50 the net cost value, N_i^{50} , used in the objective function was equal to $C_i - S_i^{50}$, since the resale value of each

Table 4.4. Net cost values of fuel saving investments

	Net cost values		
	N_i^{50}	N_i^{20}	N_i^6
Passive Solar I (per sq ft)	7.16	6.47	5.79
Passive solar II (per sq ft)	12.16	11.79	11.45
R-4 night insulation	429.00	305.91	185.30
R-6 night insulation	220.00	56.93	0
Increase 1 R-value of ceiling insulation up to R-30	2.55	0	0
Increase 1 R-value of ceiling insulation up to R-60	8.57	5.23	1.96
Increase 1 R-value of above grade insulation R-16 to R-25	3.27	-6.79	-16.64
Increase above grade insulation to R-35	666.29	630.54	595.44
Increase above grade insulation to R-45	721.05	649.54	579.41
Increase above grade insulation to R-55	796.37	689.10	584.00
Increase above grade insulation to R-65	836.96	694.99	553.80
Basement Insulation I	123.00	35.74	-49.76
Basement Insulation II	554.40	395.65	240.09
Basement Insulation III	686.40	515.36	347.76
Basement Insulation IV	819.40	576.55	338.59
Decrease air changes to .5	275.09	-195.61	-672.89
Decrease air changes to .375	500.11	-250.23	-985.35
Decrease air changes to .3	735.12	-15.11	-924.83
Decrease air changes to .25	976.00	78.99	-797.68
Insulate window headers	2.77	-6.07	-14.80
Insulate floor joist	11.06	0	0
Install a 95% AFUE furnace	4,400.00	2,200.00	1,817.00
Install a 86% AFUE furnace	2,800.00	1,400.00	1,185.00
Install a 84% AFUE furnace	2,600.00	1,300.00	1,125.00
Install a 78% AFUE furnace	2,400.00	1,200.00	1,102.00
Install a 65% AFUE furnace	2,000.00	1,000.00	1,000.00

Table 4.5. Programming solutions for alternative planning horizons

	Solution-50	Solution-20	Solution-6
Use minimum amount of south glass = 82 sq ft	x	x	x
Use maximum amount of south glass without adding additional mass = 110 sq ft			
Use a passive solar system			
Use R-4 level night insulation			
Use R-6 level night insulation			
Increase ceiling insulation to R-30 level			
Increase ceiling insulation to R-60 level	x	x	x
Increase above grade insulation to R-25		x	x
Increase above grade insulation to R-35 (change wall structure)			
Increase above grade insulation to R-45 (change wall structure)			
Increase above grade insulation to R-55 (change wall structure)			
Increase above grade insulation to R-65 (change wall structure)	x		
Basement Insulation I			x
Basement Insulation II			
Basement Insulation III			
Basement Insulation IV	x	x	
Insulate window headers	x	x	x
Insulate floor joist	x	x	x
Add 100 sq ft of nonsouth glass ^a	x	x	x
Decrease ACH to .5			
Decrease ACH to .375			
Decrease ACH to .30			x
Decrease ACH to .25	x	x	
Purchase a 95% AFUE furnace	x	x	x
Purchase a 86% AFUE furnace			
Purchase a 84% AFUE furnace			
Purchase a 78% AFUE furnace			
Purchase a 65% AFUE furnace			
Buy natural gas (amount in MBTUs)	1,379	596	233
Utilize Energy Saver Tax Credit (amount in dollars)	300	300	103
Utilize Solar Tax Credit (amount in dollars)	0	0	0

^aIndicates activity was forced into solution.

investment, R_i , is zero after 50 years (see equation 4.2). To determine how Solution 50 would be altered if the summer cooling benefits, S_i^{50} , of each investment had been ignored, the model was run using C_i as the cost coefficient in the objective function. The solution obtained is identical to Solution-50, except that Passive Solar I is included in the solution set. The additional south glass in Passive Solar I increases the cost of cooling the home in the summer. If the summer savings of each investment (which are negative for Passive Solar I) are ignored in the analysis, Passive Solar I appears to be a cost effective investment. When the summer cooling affect, S_i^{50} , is considered, however, the passive solar activity does not enter the programming solution.

The unit cost values obtained from the linear programming model's solution indicate the opportunity cost of forcing an activity into solution. The unit cost value associated with Passive Solar I in Solution-50 indicates that the total cost of heating the home would have increased by \$23 had Passive Solar I been forced into solution. Ignoring the summer cooling affects of the fuel saving activities results in a change in the solution mix that increases the total cost of heating the house over a 50 year period by \$23. The total heating cost for the 50 year period is \$15,036; therefore, the increase of \$23 represents a less than one percent increase in total heating costs. The change in total cost caused by ignoring the cooling affects of fuel saving investments does not appear to be that significant.

"Solution-6" is the cost minimizing investment mix which was obtained when it was assumed the house would be sold after six years.

The six year solution is of particular interest because six years is the average number of years that a home in Des Moines, Iowa is owned. In the six year case, the model was constrained to provide the amount of energy necessary to maintain the house at 68° F over a six year period. The energy providing coefficient, c_i , for each activity indicated the amount of energy which that activity would provide over a six year period. The net cost value, N_i^6 , of each investment was equal to the original investment cost, minus the present value of six years worth of summer cooling benefits, minus the present value of the resale value of the investment (assuming that it is sold after six years) i.e., $N_i^6 = C_i - S_i^6 - R_i^6$.

As indicated in Table 4.5, Solution-6 differs from the 50 year solution in several ways. In both Solution-50 and Solution-6, the maximum energy efficient furnace enters the solution. However, the amount of insulation installed is less, and the air changes per hour are greater, in the six year solution. In Solution-50, all insulation levels enter at their maximum levels. In Solution-6, an R-25 level of above grade wall insulation is installed, rather than the maximum level of R-65. Basement Insulation I enters Solution-6, as opposed to Basement Insulation IV in Solution-50. Basement Insulation I is the least costly and least extensive basement insulation alternative considered. The level of air changes per hour (ACH) in the six year solution is .30 ACH as opposed to .25 ACH in the 50 year solution.

The comparison of the six and 50 year solutions indicates that when total cost is minimized under the assumption that the home is to be sold in six years there is an underinvestment in fuel saving activities relative to the long run cost minimization solution (Solution-50). An examination of the unit cost values associated with the activities that enter Solution-6 but not Solution-50 (Basement Insulation I, .30 ACH, and R-25 wall insulation) indicate that the total cost of heating the home over a 50 year period would have increased by over \$1,900 had these activities been forced into Solution-50. Although Solution-6 indicates the investment mix which will minimize total cost for an individual who plans to own the home for a six year period, this investment mix will increase the long-run total cost of heating the home by over 12 percent.

For comparison purposes, a six year solution was obtained when resale values were ignored. In this case, the cost coefficient used was $C_i - S_i^6$. R_i^6 was assumed equal to zero. This solution differs from Solution 6 in several ways. The level of ceiling insulation is reduced from R-60 to R-30, the level of air changes per hour is increased from .3 to .5 ACH, and Basement Insulation I drops out of solution. These three changes result in a 20 percent increase in the total cost of heating the home over a six year period (an increase of \$374). If resale values are not considered when determining the "optimal" investment mix, there is a substantial increase in the total cost of heating the home.

Finally, the investment mix which minimizes total cost under the assumption the house is to be sold after twenty years was obtained. This investment mix is entitled "Solution-20" on Table 4.5. Solution-20

differs from Solution-50 only in the level of above grade wall insulation which is installed. In Solution-20, the level of wall insulation which enters is R-25, as opposed to R-65 in Solution-50. At this lower level of wall insulation, the total cost of heating the home over a 50 year period would increase by \$743, a five percent increase in total cost. When resale values were ignored in the twenty year model (i.e., $C_i - S_i^{20}$ was used as the cost coefficient) only one change occurred. In Solution-20, a .25 ACH level enters the solution mix. When resale values were ignored, however, the ACH level increased to .30 ACH. This change in the investment mix increases the total cost of heating the home over a 20 year period by \$137. This represents a two percent increase in total cost.

In this section, the three programming solutions obtained have been discussed and compared. It was found that when total cost is minimized over a fifty year period, all of the fuel conserving activities, except for the passive solar activities, enter the solution mix at their maximum levels. Examining Solution-6 revealed that the level of fuel conserving investments undertaken would decrease when total cost is minimized under the assumption that the house is to be sold in six years. This reduction in the level of fuel conserving investments caused a 12 percent increase in the total cost of heating the house over a fifty year period. When total cost was minimized under the assumption that the house is to be sold in 20 years (Solution-20), the level of fuel conservation was less than Solution-50 but greater than Solution-6. At this level of

conservation investment, the long-run total cost of heating the house was five percent higher than it is in Solution-50.

These results reveal that the "efficient" investment for a given household is a function of the number of years that it plans to own the home. Although Solution-50 indicates the investment mix which minimizes the long run total cost of heating the home, this investment mix is not optimal for a household that only plans to own the home for a 6 or 20 year period. No single "optimal" investment mix may be defined; rather, what is optimal varies according to the length of time the household plans to own the home.

Finally, the impact of ignoring the resale value of fuel saving investments was examined in both the 6 and 20 year model. In the six year model, the impact of ignoring resale values was significant. When resale values were excluded from the analysis an underinvestment in fuel conservation activities resulted which increased the total cost of heating the home over a six year period by 20 percent. In the 20 year model, the impact of ignoring resale values was much less substantial. In this case, the underinvestment caused by the exclusion of resale values resulted in only a two percent increase in the total cost of heating the home over a 20 year period. It is concluded, therefore, that accurate knowledge of the resale values of fuel saving investments is necessary for households planning to own the home for a period of time less than the life of the fuel saving investments. Excluding these resale values from the investment decision will increase the total cost to heating the home over the period of time it is owned. The shorter the

time period that the home is to be owned, the greater the excess heating cost incurred by excluding resale values from the investment decision.

Sensitivity Analysis

Temperature sensitivity analysis

As discussed in the theory chapter, a weakness in using a linear programming approach to determine the "optimal" fuel saving investment mix is that the temperature setting of the home is exogenous to the model. The solution obtained from the linear programming model, therefore, is truly optimal only if the temperature used in the model is the utility maximizing temperature of the homeowner. To explore the ramifications of this weakness, sensitivity analysis was undertaken to determine how sensitive the linear programming model solutions are to changes in temperature settings, i.e., if the temperature assigned in the linear programming model is different than the utility maximizing temperature of the homeowner, will the optimal fuel saving investment mix be substantially different?

To explore this question, solutions were obtained for both the 6 and 50 year models under the assumption that the temperature was maintained at 64° F and at 72° F, alternatively. In each case, there was no change in the optimal fuel conserving investment mix. The only solution changes resulting from the new temperature constraints were variations in the quantities of natural gas purchased in order to maintain the different temperatures. These results reveal that the investment solutions

indicated on Table 4.5 are "optimal" over the range of utility maximizing temperatures between 64° F and 72° F. The stability of these solutions with respect to temperature variations minimizes the weakness caused by the exogeneity of the internal temperature setting specified in the linear programming model.

Fuel Price Sensitivity Analysis

As discussed in the first section of this chapter, a limitation of the linear programming model is that it cannot forecast price expectations. In the home heating linear programming model used in this paper, a key assumption is made concerning the future price trend of natural gas. As stated previously, a five percent real rate of fuel price increase was assumed. If the actual real rate of fuel price increase which occurs is greater than five percent then the model's solution will indicate an underinvestment in fuel conserving activities. Conversely, if the actual fuel price increase is less than five percent, the model's solution will exhibit an overinvestment in fuel saving activities. A change in the price trend would, of course, affect the 50 year solution more than the 20 or six year solutions.

To test how sensitive the optimal activity mix is to changes in price trends, the price of natural gas was parameterized in the 50 year model and solutions were obtained at alternative real rates of price increase. Table 4.6 summarizes the optimal activity mixes obtained under alternative price increase assumptions. The initial model (Solution-50) was run using a five percent real rate of fuel price increase. The

Table 4.6. Programming solutions for alternative planning horizons

	0%	(1%-4%)	5%	(6%-8%)	9%
Use minimum amount of south glass = 82 sq ft	x	x	x		
Use maximum amount of south glass without adding additional mass = 110 sq ft				x	
Use a passive solar system					x
Use R-4 level night insulation					
Use R-6 level night insulation				x	x
Increase ceiling insulation to R-30 level					
Increase ceiling insulation to R-60 level	x	x	x	x	x
Increase above grade insulation to R-25	x	x			
Increase above grade insulation to R-35 (change wall structure)					
Increase above grade insulation to R-45 (change wall structure)					
Increase above grade insulation to R-55 (change wall structure)					
Increase above grade insulation to R-65 (change wall structure)			x	x	x
Basement Insulation Possibility I	x				
Basement Insulation Possibility II					
Basement Insulation Possibility III					
Basement Insulation Possibility IV		x	x	x	x
Insulate window headers	x	x	x	x	x
Insulate floor joist	x	x	x	x	x
Add 100 sq ft of nonsouth glass ^a	x	x	x	x	x
Decrease ACH to .5					
Decrease ACH to .375					
Decrease ACH to .30	x				
Decrease ACH to .25		x	x	x	x
Purchase a 95% AFUE furnace	x	x	x	x	x
Purchase a 86% AFUE furnace					
Purchase a 84% AFUE furnace					
Purchase a 78% AFUE furnace					
Purchase a 65% AFUE furnace					
Buy natural gas (amount in MBTUs)	1,908	1,317	1,124	1,051	993
Utilize Energy Saver Tax Credit (amount in dollars)	104	208	300	300	300
Utilize Solar Tax Credit (amount in dollars)	0	0	0	88	88

^aIndicates activity was forced into solution.

parameterization of natural gas prices indicates that if the actual real rate of fuel price increase is four percent or less, the optimal level of wall insulation decreases from R-65 to R-25. No other fuel saving investments are altered until the rate of fuel price increase is reduced to zero. If real fuel prices are constant over time, then Basement Insulation I enters the solution rather than Basement Insulation IV and the optimal level of air changes per hour increases from .25 to .30. If the actual rate of fuel price increase is six percent or greater, then Passive Solar I, in which the amount of south glass is increased up to ten percent of the total floor area, enters the solution set. This solution is stable until the annual real rate of price increase is nine percent. At this rate of price increase, Passive Solar II (i.e., south glass is increased up to 20 percent of the total floor area and additional mass is added to the home) enters the solution.

In general, the fuel price sensitivity analysis indicates that the majority of the activities in the model are stable with respect to fuel price variation. Between the range of one and eight percent rate of annual price increase, only the amounts of wall insulation and south glass are altered, all other activities are stable.

It is interesting to note the order in which activities entered the solution as progressively higher real rates of price increase were assumed. Ceiling, window header, and floor joist insulation entered at their maximum levels, and a 95 percent AFUE furnace entered the solution even when the price of fuel was assumed to be constant over time. These investments will prove to be cost effective when the future rate of fuel

price increase is quite uncertain, but positive. A high level of basement insulation and tight construction were also shown to be cost effective at rates of fuel price increase far less than the five percent rate assumed. These investments are also relatively "safe" investments. The activities of changing the wall structure in order to install high levels of wall insulation, and increasing the amount of south glass to ten percent of the total floor area were cost effective only if the rate of price increase was 5 and 6 percent, respectively. Finally, Passive Solar II is the least likely to prove to be cost effective. Only if the annual rate of price increase is greater than or equal to nine percent will Passive Solar II decrease the total cost of heating the home.

In this chapter, the cost minimization linear programming model used to obtain the cost minimizing fuel saving investment mix under alternative assumptions was examined. The solutions obtained under the assumptions that the house is to be owned for 6, 20, and 50 years were discussed and compared. When heating costs were minimized over a 50 year period all of the conservation investments entered the solution at their maximum levels except for the passive solar activities. When costs were minimized under the assumption that the home was to be sold after six years, the level of conservation investment decreased. The solution in this case minimized the total heating cost of the six year homeowner, however, it resulted in a 12 percent increase in the total cost of heating the home over a 50 year period. The solution obtained when costs were minimized under the assumption that the home would be sold after 20 years only differed from the 50 year solution in the level of wall

insulation that was incorporated into the house structure. The lower level of wall insulation included in the 20 year solution resulted in a five percent increase in the total cost of heating the home over a 50 year period. These results revealed that the "optimal" fuel saving investment mix varies according to the length of time the home is to be owned.

The effect of excluding resale values from the 6 and 20 year models was examined. It was found that when the house is to be sold after six years, the total cost of heating the home over that period is increased substantially (by 20 percent) when resale values are excluded from the investment decision. Ignoring resale values in the investment decision has little effect, however, when it is assumed that the home will be owned for 20 years. The total cost of heating the home over the 20 year period increased by only two percent when resale values were excluded from the investment decision. The value of accurate knowledge of resale values, therefore, increases as the length of time the home is to be owned decreases.

Sensitivity analysis was performed to determine how the cost minimizing solution mix for the 50 year model would change as the temperature level specified in the linear programming model was altered. Solutions were obtained under the assumption that the house was maintained at 64° F and 72° F, respectively. The level of conservation investment in these two solutions was identical to Solution-50, in which it was assumed that the home was to be maintained at 68° F. These results reveal that the level of conservation investment obtained from

the linear programming model is "optimal" over a fairly wide range of temperature variation.

Finally, sensitivity analysis was undertaken to determine how the cost minimizing investment mix would change under alternative assumptions concerning the future annual rate of price increase. In obtaining Solution-50, a five percent real rate of price increase was assumed. Sensitivity analysis revealed that the "optimal" solution is altered under alternative real rates of price increase. However, many of the investment levels are stable over a wide range of rates of price increase. Between a 1% and 8% real rate of price increase, the level of ceiling, basement, window header and floor joist insulation, and the level of air changes per hour in the home were constant; only the level of wall insulation and the amount of south glass were altered.

By examining the order in which conservation activities entered the solution mix as progressively higher rates of price increase were assumed, the relative cost effectiveness of investments were examined. The maximum levels of ceiling, window header, and floor joist insulation proved to be cost effective as long as real prices were not decreasing over time. The maximum level of basement insulation and tight construction also proved to be cost effective at rates of price increase far less than the five percent real rate assumed in the initial analysis. Passive Solar I and Passive Solar II were the least likely to be cost effective. Passive Solar I decreased the total cost of heating the home over a 50 year period only when the rate of price increase assumed was six percent or greater and Passive Solar II decreased the total cost of

heating the home only when a nine percent real rate of price increase was assumed.

In the following chapter, the policy implications of the linear programming model results and the results of the hedonic price model will be explored.

CHAPTER 5. CONCLUSIONS AND POLICY IMPLICATIONS

In this study, a hedonic model was used to determine how energy efficiency is evaluated in the Des Moines housing market. The relationship between housing prices and characteristics was estimated using a Box-Cox model and the implicit price of each individual characteristic was found by differentiating the hedonic price function with respect to the characteristic, *ceteris paribus*. This process revealed that a premium is paid for energy efficient homes in Des Moines. The implicit price of energy efficiency obtained indicates that, on average, a \$1 decrease in annual fuel expenditures (due to an increase in efficiency) increases the expected selling price of the house by \$11.63.

Since the hedonic price function is nonlinear, the implicit price of increases in energy efficiency will vary with the level of efficiency itself and the level of other housing characteristics. By evaluating the second derivatives of the hedonic price function, it was revealed that the implicit price of energy efficiency is greater than \$11.63 for relatively inefficient homes (i.e., homes having high fuel expenditures per square-foot) and less than \$11.63 for relatively efficient homes. The derivative of the implicit price with respect to age indicates that the implicit price of increases in energy efficiency is inversely related to the age of the house. The premium paid for energy efficiency in older homes is less than the premium paid for energy efficiency in newer homes.

The implicit price of energy efficiency obtained from the hedonic price model was used to calculate the resale values of fuel saving investments. These resale values were incorporated into a cost minimizing linear programming model. The linear programming model generated an efficient energy providing investment mix for a baseline home under the assumption that it was to be owned for 50, 20, and 6 years, alternatively. In the 50 year model, the initial investment cost of each activity was weighed against the present value of the entire flow of marginal benefits generated by the investment. In the six and 20 year models, the initial investment cost of each activity was weighed against the present value of benefits the investment generated during the period the home was owned and the resale value of the investment. In order to obtain a solution mix for the six and 20 year linear programming models, therefore, it was necessary to be able to estimate the resale value of each fuel saving investment. The implicit price of fuel savings obtained from the hedonic price model was used to generate these resale values.

A question of interest is: Is the Des Moines housing market pricing fuel savings (i.e., energy efficiency) efficiently? As previously mentioned, the hedonic price model reveals that a premium is paid for energy efficient homes. On average, a \$1 decrease in energy expenditures is expected to increase the selling price of the home by \$11.63. Is this premium "correct", or is the housing market undervaluing or overvaluing fuel savings? By examining the three linear programming solutions, the answer to this question may be explored. Solution-50 indicates the investment mix which minimizes the total cost of heating

the home over a 50 year period. The 20 and six year resale solutions (Solution-20 and Solution-6) both reveal a lower level of conservation investment than Solution-50. The solution obtained under the assumption that the house will be sold after 20 years (Solution-20) indicates that the level of wall insulation installed is less than in Solution-50. This decrease in insulation results in a five percent increase in the cost of heating the home over a fifty year period. The solution obtained under the assumption that the home was to be sold after six years (Solution-6), indicates that the levels of wall and basement insulation are less than in Solution-50, and the level of air changes per hour is greater. This reduction in the level of conservation investment increases the total cost of heating the home over a fifty year period by 12 percent.

At first glimpse, it might be thought that these results indicate that the housing market is undervaluing fuel saving investments, since the resale solutions (Solution-6 and Solution-20) indicate an underinvestment in conservation activities relative to the long-run cost minimization solution (Solution-50). However, these results do not necessarily indicate that the market is pricing fuel saving investments inefficiently. The difference between Solution-50 and the conservation investment levels obtained in Solution-6 and Solution-20 may be due to several factors.

Theoretically, the resale value of each fuel saving investment is equal to the present discounted value of the fuel savings which will be obtained from it. In calculating the present value of fuel savings, home buyers in Des Moines may not use the five percent real rate of fuel price

increase and the five percent real discount rate that are assumed in this paper. If the resale values obtained in this analysis reflect a less than five percent expected real rate of price increase, or a greater than five percent real discount rate utilized by Des Moines home buyers, then it would be expected that Solution-6 and Solution-20 would utilize a lower level of fuel conservation than Solution-50, in which all fuel savings are evaluated using a five percent expected rate of real price increase and a five percent real discount rate. This cause for a discrepancy between Solution-50 and the six and 20 year resale solutions does not indicate that the housing market is failing to price fuel saving investments efficiently and would not necessitate any policy action.

A second potential source of the difference in conservation investment levels in the three solutions is the error component in the method used to calculate the resale value of fuel saving investments. As discussed in Appendix C, there are several inherent limitations in the method used to calculate the resale value of fuel saving investments. If better information on the actual resale values of fuel saving investments was incorporated into the six and 20 year linear programming models, the solutions may reveal that the levels of conservation investment are closer to the conservation investment level obtained in Solution-50. Conversely, use of better information on actual resale values may also reveal that the optimal level of conservation investment is less than the levels indicated in Solution-20 and Solution-6.

A third possible explanation for the discrepancy in investment levels among the three solutions is the existence of a market failure.

It is possible that the conservation levels indicated in Solution-6 and Solution-20 are less than in Solution-50 because the housing market is not pricing fuel saving investments efficiently. If Des Moines home buyers are not aware of the level of energy efficiency of homes on the resale market, then it cannot be expected that this information will be incorporated into housing prices. By improving the information about the relative efficiency of homes on the housing market, it may be possible to increase the efficiency of the housing market in attaching premiums to homes which are highly energy efficient. The Iowa Extension Service at Iowa State University has designed an efficiency index called the Home Heating Index (Hodges et al., 1982). This index is a measure of the efficiency of the house structure and does not reflect the lifestyle of the occupants of the home. It has been suggested that all of the homes on the housing market be ranked according to the Home Heating Index. This ranking would enable consumers to easily compare the relative efficiency of homes, just as they can compare miles per gallon when purchasing a car. An increase in information available to home purchasers may increase the efficiency of the housing market in attaching premiums to energy efficient homes.

Since any one of these three possible explanations may account for the difference between Solution-50 and the two resale case solutions, it cannot be concluded that the market is not pricing fuel savings efficiently. Further information about the private discount rates and fuel price expectations of Des Moines home buyers is necessary in order to determine if the housing market is pricing fuel savings efficiently.

Consider a home which is 30 years old, the average age of homes in the Des Moines sample. Assume that an investment in this 30 year old home caused a \$1 annual savings and was expected to last for another 20 years. Evaluated using the five percent real discount rate and five percent real rate of price increase assumed in the linear programming model, the present value of the remaining \$1/year savings caused by this investment would be \$20. According to the hedonic price model, however, this \$1/year savings will only increase the resale value of the average aged home by \$11.63. It is possible that this \$11.63 implicit price does not reflect a failure of the market, but merely a difference in the discount rate and price expectations used by Des Moines home buyers. Without better information concerning the price expectations and private discount rates utilized, it may not be concluded that the Des Moines housing market is, or is not, pricing fuel saving investments efficiently.

Knowledge of the implicit price of energy efficiency does, however, provides valuable information for households which do not plan to own their homes over the entire life of the fuel saving investments. Information on the implicit price of energy efficiency enables each household to estimate the resale value of fuel saving investments and incorporate these resale values into their investment decisions. The solution obtained when resale values were excluded from the six year linear programming model indicates the importance of this information. When resale values were set equal to zero in the six year linear programming model, the solution mix revealed an underinvestment in fuel saving activities. This decrease in the level of conservation caused the

total cost of heating the home over a six year period to be 20 percent higher than the total cost in Solution-6, in which resale values are included. Likewise, if resale values had been overestimated, overinvestment in fuel saving activities would have occurred and total cost would have been greater than the total cost incurred in Solution-6. Accurate knowledge about the resale value of fuel saving investments, therefore, will enable consumers to minimize their total heating costs. More research on the resale value of fuel saving investments and dissemination of this information to the public will enable homebuilders to better incorporate resale values into their investment decisions.

The results of the linear programming model reveal that the cost minimizing fuel saving investment mix for a given home varies according to the length of time the home is to be owned. No single "optimal" investment mix exists. The model results also indicate that the "optimal" investment mix varies according to the future rate of natural gas price increase that is assumed. As the annual rate of price increase assumed rose for 0-9 percent, the level of conservation investment increased, and the amount of natural gas consumed decreased by nearly 50 percent. Since the level of conservation investment is sensitive to the future rate of natural gas price increase, government policies which cause the price of natural gas to be arbitrarily low will cause an underinvestment in conservation activities, while government policies

which cause the rate of price increase of natural gas to be arbitrarily high, will cause an overinvestment in conservation activities.¹

Two of the activities in the linear programming model involve utilizing tax credits. The Energy Saver Tax Credit is a 15 percent tax credit which is available for investments in insulation, and a 40 percent Solar Tax Credit is available for single purpose solar investments. The amount of each tax credit which may be utilized is constrained to reflect the Internal Revenue Service (IRS) regulations. The maximum total Energy Saver Tax Credit which is allowed by IRS regulation is \$300, while the maximum allowable level of the Solar Tax Credit is \$4,000. Examination of the long run solution of the linear programming model (Solution-50) reveals the impact of these restrictions. The level of insulation investment in Solution-50 creates \$358 worth of potential Energy Saver Tax Credit, however, only \$300 worth of this tax credit is allowed to enter the solution. If the IRS was to increase the allowable Energy Saver Tax Credit to an amount greater than or equal to \$358, therefore, the consumer's total heating cost would be reduced.

Solution-50 reveals that no amount of the Solar Tax Credit is utilized. The penalty cost associated with the \$4,000 maximum allowable level, therefore, is zero. If the maximum level of the Solar Tax Credit was raised or reduced, the total cost of heating the home would be unaltered. The level of Solar Tax Credit utilized in the model is

¹Holding the rate of natural gas price increase below market value will act as a tax on conservation activities, while holding the rate of natural gas price increase above market value will have the same effect as a subsidy on conservation activities.

obviously zero, since neither of the two passive solar activities enter the programming solution. The Solar Tax Credit could not be applied to the passive solar activities, however, even if they had entered the solution. The Solar Tax Credit has a restriction specifying that it is not applicable to solar components serving a dual purpose (E. Roach. Internal Revenue Service, Washington, D.C., personal communication, 1981). Passive solar systems are typically an integral part of the house structure, with their components serving the dual purpose of a floor or wall. Due to the dual purpose restriction, the Solar Tax Credit is not applicable to the increased glass area and increased mass area required for a passive solar system, and neither of the two passive solar activities in the model qualify for the tax credit.

Examining the lower cost values associated with Passive Solar I and Passive Solar II reveal the impact which relaxing the dual purpose restriction would have on the fuel saving investment mix. The lower cost figure for an activity indicates the cost coefficient necessary to bring that activity into solution without a penalty cost. The lower cost figure associated with Passive Solar I (increasing south glass up to ten percent of the total floor area without adding additional mass) is \$6.82. The net cost coefficient for Passive Solar I in the 50 year model is \$7.16. Out of the net cost, \$5 represents the initial investment cost and \$2.16 represents the present value of the additional summer cooling cost due to the additional south glass. If the 40 percent Solar Tax Credit could be applied to the initial investment cost, the net cost coefficient for Passive Solar I would be reduced to \$5.16. Since this

cost coefficient is less than the lower cost figure of \$6.82, Passive Solar I would enter the optimal solution mix. If the dual purpose restriction was dropped, therefore, Passive Solar I would be a part of the cost minimizing fuel saving investment mix.

The lower cost figure associated with Passive Solar II (increasing south glass from ten percent to 20 percent of the total floor area and adding additional mass to the house structure) reveals that the cost coefficient for Passive Solar II would have to decrease to \$3.67 for the activity to enter with zero penalty cost. The cost coefficient for Passive Solar II is \$12.16, with \$10 representing the initial investment cost and \$2.16 representing the present discounted value of the increased summer cooling cost due to the additional south glass. If the 40 percent Solar Tax Credit could be applied to the initial investment cost, the cost coefficient would be reduced to \$8.16. At this reduced level, however, the cost coefficient is still greater than the \$3.67 value necessary for Passive Solar II to enter the solution with zero penalty cost. Dropping the dual purpose restriction on the Solar Tax Credit, therefore, would cause Passive Solar I to enter solution but a further subsidy would be necessary for Passive Solar II to enter the linear programming solution.

In conclusion, this study reveals that a premium is obtained for energy efficient homes in Des Moines. On average, a \$1 decrease in annual fuel expenditures will increase the expected selling price of the house by \$11.63. Without further information, however, no conclusions

may be drawn as to whether this implicit price indicates that the housing market is pricing fuel saving investments efficiently.

Based on the implicit price of energy efficiency, the resale value of fuel saving investments was estimated. Using these resale values, a linear programming model generated a cost minimizing fuel saving investment mix for a baseline home under the assumption that it was to be owned for 50, 20, and 6 years, alternatively. This process revealed that no single investment mix is "optimal." Rather, the efficient investment mix is a function of the period of time which the home is owned, the future rate of price increase assumed, and the tax policies instituted.

Further research on the resale value of fuel saving investments and dissemination of this information to the public would aid households in choosing an efficient fuel saving investment mix. Better information true structural efficiency of homes would augment these research efforts.

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APPENDIX A

Table A.1. Obtaining the relationship between annual auxiliary heat requirement and model activities

Activity	Number of observations	Slope coefficient ^a	Relevant range
Increasing ceiling insulation	9	-0.40126 -0.10464	R15-R33 R33-R55
Increasing above-grade wall insulation	15	-0.34290 -0.12190 -0.06510	R12-R25 R25-R39 R39-R52
Increasing south glass	10	-0.02160 -0.01165	42-110 sq ft 110-208 sq ft
Increasing nonsouth glass	7	0.08712	0-100 sq ft
Using R6 night insulation	6	-0.04645	42-208 sq ft
Using R4 night insulation	6	-0.03506	42-208 sq ft

^aChange in the annual auxiliary heat requirement brought about by a one unit change in the activity.

Table A.2. MBTUs provided by activities analyzed on a nonincremental basis

Activity	MBTUs provided ^a
Basement Insulation Possibility I	2.734
Basement Insulation Possibility II	4.974
Basement Insulation Possibility III	5.359
Basement Insulation Possibility IV	7.609
Insulate window headers	0.277
Insulate floor joist	2.019
Decrease ACH to .500	18.886
Decrease ACH to .375	23.506
Decrease ACH to .300	26.269
Decrease ACH to .250	28.105

^aMBTUs provided per heating season.

APPENDIX B

Calculating the Value of Summer Cooling Benefits
Associated with Fuel Saving Investments

Investments which are installed to increase the heating efficiency of the home may also reduce the cost of cooling the home in the summer. In order to incorporate these cooling benefits into the investment decision, it is necessary to estimate the reduction in annual summer cooling costs which each investment brings about. In calculating the summer cooling benefits, it is assumed that the home is maintained at 78° F throughout the cooling season. The weather conditions used are those corresponding to an average cooling season in Des Moines, Iowa.

The cooling benefits associated with increases in insulation levels are estimated by calculating the cooling costs necessary due to heat gain through the insulated surface, assuming alternative insulation levels. As the amount of insulation covering a surface increases the heat gained through the surface decreases, and the amount of energy needed to rid the house of that excess heat is less. By calculating the cost of ridding the home of the excess heat gained through a surface covered by alternative insulation levels, the impact of changes in insulation levels on cooling costs may be obtained.

The formula used to calculate the cost of cooling necessary to rid the home of excess heat gained through an insulated surface is:

$$\text{\$ cost} = \frac{\text{Hg} \times \text{A} \times \text{P}}{1000 \times \text{R} \times \text{EER}} \quad (\text{B.1})$$

where: H_g = BTUs of heat gain through the surface,

A = area of surface in square feet,

P = price of electricity (= \$.07 per kwh),

R = R-value of insulation, and

EER = Energy Efficiency Ratio of cooling system (assumed = 9).

Using this cost formula, the savings due to increases in insulation may be obtained. For example, the cost of cooling which is necessary due to heat gain through the ceiling is \$12.40, assuming that an R-15 level of insulation is in the ceiling. If the amount of insulation is increased to R-20, there is a reduction in the amount of heat gained through the ceiling, and therefore a decrease in the cooling cost necessary to rid the house of excess heat gained through the ceiling. The increase from R-15 to R-20 insulation reduces the annual cooling expenditure by \$3.10, or \$.62 per R-value.

Cost formula A.1 can be used to calculate the summer cooling savings due to additional insulation in the ceiling, above grade walls, floor joist and window headers.

The cooling costs of the home are a function of the air changes per hour in the home as well as the heat gained through the above ground surfaces. By calculating the cooling costs which are created by air infiltration into the home (assuming alternative air changes per hour), the decrease in cooling expenditures created by decreases in air changes per hour may be estimated. The cost formula used to calculate the cooling costs created by air infiltration at various ACH levels is:

$$\text{\$ cost} = \frac{\text{DH} \times \text{ACH} \times \text{V} \times .018 \times \text{P}}{1000 \times \text{EER}} \quad (\text{B.2})$$

where: DH = degree hours (=6,255),

ACH = air changes per hour,

V = volume of house in cubic feet,

P = price of electricity, and

EER = Energy Efficiency Ratio.

Finally, the cooling costs of the home are a function of the amount of south glass in the home. As the amount of south glass increases the amount of heat gained increases and the cost of ridding the home of this excess heat increases. By calculating the cost of ridding the home of the excess heat gained through alternative amounts of south glass, the increase in cooling costs caused by increases in south glass may be obtained. The formula used to calculate the cooling cost due to heat gain through alternative amounts of south glass is:

$$\text{\$ cost} = \frac{\text{Hg} \times \text{A} \times \text{P}}{1000 \times \text{EER}} \quad (\text{B.3})$$

Since cooling costs increase as more south glass is added to the home the "savings" associated with increases in south glass are negative.

Once the annual savings associated with each investment are estimated by the method out-lined above, the present discounted value of these savings may be obtained. The formula used to calculate the present value of the annual savings associated with each investment is:

$$PV_i = S_i \frac{1 - (1+d-r)^{-t}}{d - r} \quad (B.4)$$

where: PV_i = present value of savings due to investment i ,

S_i = annual savings of investment i ,

d = discount rate (assumed = 5 percent),

r = rate of price increase (assumed = 0.5 percent) (Office of Planning and Analysis, 1983), and

t = time horizon.

Table B.1 has the present discounted value of 6, 20, and 50 years worth of annual summer savings associated with the fuel saving activities used in the linear programming model. It is assumed that the furnace, basement insulation and night insulation do not create significant summer savings.

Table B.1. Summer savings caused by fuel saving investments

Activity	Annual summer savings	Present value of t year's worth of summer savings:		
		t=50	t=20	t=6
Increase ceiling insulation by 1 R-value in R-15 to R-30 range	\$.41	\$ 8.10	\$ 5.33	\$ 2.11
Increase ceiling insulation by 1 R-value in R-30 to R-60 range	.10	1.98	1.30	.52
Increase wall insulation by 1 R-value in R-15 to R-25 range	.16	3.16	2.08	.83
Increase wall insulation from R-25 to R-35	.68	13.44	8.84	3.51
Increase wall insulation from R-25 to R-45	1.06	20.95	13.79	5.46
Increase wall insulation from R-25 to R-55	1.31	25.89	17.04	6.76
Increase wall insulation from R-25 to R-65	1.47	29.05	19.12	7.58
Insulate window headers	.16	3.16	2.08	.83
Insulate floor joist	.84	16.60	10.92	4.33
Decrease ACH from 1 to .5	5.47	108.10	71.15	28.21
Decrease ACH from 1 to .375	6.84	135.17	88.97	35.28
Decrease ACH from 1 to .30	7.66	151.38	99.64	39.51
Decrease ACH from 1 to .25	8.21	162.24	106.79	42.35
Add 1 sq ft of south glass	-.12	-2.37	-1.56	-.62

APPENDIX C

Discussion of Resale Values Obtained from the Implicit
Price of Energy Efficiency

As explained in the data section of Chapter 4, the resale values obtained for fuel saving investments are based on the implicit price of energy efficiency (i.e., fuel savings). The underlying assumptions of the implicit price, therefore, must be kept in mind when interpreting the resale value of each fuel saving investment. Four issues which must be considered when interpreting the implicit price derived from the hedonic model and the resulting resale values which are obtained are discussed here:

(1) The functional form of the hedonic model is not linear; therefore, the implicit price of energy efficiency is not constant. The change in the price of the house brought about by a change in fuel expenditures depends on the initial level of fuel expenditures as well as the level of other characteristics of the home. The implicit price which is used to estimate the resale value of each fuel saving investment is calculated under the assumption that the level of predicted fuel bills per square foot, \hat{F}^* , and all other housing characteristics are equal to the mean values in the Des Moines housing sample. The actual implicit price of fuel savings will vary if the housing characteristics and the value of \hat{F}_i^* are not equal to their mean values. Since the second derivative of the hedonic price function with respect to \hat{F}^* is positive,

the implicit price of fuel savings will be greater than 11.63 in inefficient homes and less than \$11.63 in efficient homes.

(2) The implicit price a buyer is actually willing to pay for a dollars worth of fuel savings depends on the age and remaining life of the fuel saving investments in the home, as well as expected rate of natural gas price increase and the rate at which future savings are discounted. The implicit price obtained from the hedonic model represents an average of the premiums paid for homes having fuel saving devices of varying ages. The implicit price is most accurate, therefore, when it is assumed that the fuel saving investments are the average age of all fuel saving investments in the Des Moines housing sample.

The average age of the fuel saving investments in the sample of homes, however, is not known. The average age of the homes in the sample is 30 years old. If all of the fuel saving devices were installed when the homes were built, and not replaced, then their average age would be 30 years also. A more realistic possibility is that the mean age of the fuel saving devices in the housing sample is not equal to the mean age of the homes. The increase in fuel prices in the early 1970s caused a new concern for fuel conservation and it is possible that many of the fuel saving investments in homes were installed since that time. Investments such as adding ceiling insulation and calking windows are easily added to existing homes. In addition, insulation may be blown into existing walls, solar retrofitting is possible and high energy efficiency furnaces may be purchased. Therefore, the mean age of fuel saving investments may be different than the mean age of the houses in the study. Since there

is no information available concerning the mean age of the fuel saving investments in the housing sample this figure had to be approximated.

Taking into account that the mean age of the houses in the sample is 30 years and that furnaces may have been replaced, caulking and insulation may have been added, and solar retrofitting may have occurred, it is hypothesized that the mean age of the fuel saving investments in the sample is 20 years. If half of the fuel saving investment in the home had been installed at the time the home was built and the second half had been installed during the period of rising fuel prices in the early 1970s, then the hypothesis would hold.

Based on the hypothesis that the mean age of fuel saving investments in the Des Moines sample is 20 years old, the implicit price of energy efficiency reflects the premium paid for an increase in efficiency in a home in which the fuel saving investments are 20 years old. Since the resale value of each fuel saving device is based on the implicit price of energy efficiency, the resale values are also thought to be most accurate when they are calculated under the assumption that each fuel saving device is 20 years old.

(3) In calculating the resale values according to the method outlined in Chapter 4, it is apparent that each resale value is based on the winter fuel savings of the investment; potential summer savings are not included. For example, in obtaining the resale value of increased ceiling insulation, the reduction in annual heating expenditures the additional insulation brings about is multiplied by the implicit price of fuel savings obtained from the hedonic model. This figure is then used

as the resale value of ceiling insulation. The amount of summer savings associated with the increased insulation is not considered when determining its resale value.

This method of basing the resale value of each investment only on its winter savings rests on the assumption that the premium buyers are willing to pay for an efficient home does not reflect the value of any summer savings which may be obtained due to the increased winter efficiency. In other words, it is assumed that the implicit price of increases in winter heating efficiency obtained from the hedonic model reflects only the expected value of the decrease in winter fuel expenditures.

This assumption is made for three reasons. First, the summer savings associated with investments which increase the winter efficiency of the home are difficult to calculate. They are highly variable depending on the temperature at which the home is maintained, the amount of cooking which takes place in the home, the outside temperature level and the humidity. Second, when the cooling benefits of fuel saving investments are approximated by the method outlined in Appendix B, they prove to be very small compared to the heating benefits of the investments. For example, as the amount of wall insulation is increased from R-25 to R-65, the decrease in annual winter fuel expenditures is \$26. The annual summer cooling benefits associated with this investment are only \$1.47, less than six percent of the winter savings. Finally, the results of the hedonic model itself do not indicate that the implicit price of energy efficiency obtained is a function of the summer cooling

benefits which may be incurred. When the hedonic model was initially constructed, the fuel bills variable, \hat{F}^* , was interacted with the air-conditioning variable, CA (i.e., the term $\hat{F}^* \times CA$ was included in the regression). It was hypothesized that if home purchasers include summer cooling effects in the premium that they are willing to pay for winter efficient homes then the coefficient on this interaction term would be negative and significant, i.e., home purchasers using summer cooling would be willing to pay a higher premium for energy efficient homes than buyers purchasing homes which do not have summer cooling. It was found, however, that the coefficient on the fuel expenditure-air-conditioning term was not significant.

(4) A final point is that the retrofitting cost of each investment provides an upper limit on its resale value. For example, when purchasing a home having a high efficiency furnace the buyer would not be willing to pay a premium for the home which is higher than the cost of having the furnace installed after the house is purchased. In cases where the retrofitting cost would be equivalent to the original installment cost, the original installment cost was used as an upper limit. This restriction was used for the following activities:

- 1) adding ceiling insulation, 2) insulating the floor joist, and
- 3) installing night insulation on the south glass.